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ABSTRACT

This book contains 65 physics experiments. The experiments are for a college-level physics course for music and art majors. The initial experiments are devoted to the general concept of vibration and cover vibrating strings, air columns, reflection, and interference. Later experiments explore light, color perception, cameras, mirrors and symmetry, polarization, the response of the eye and ear to intensity changes, and other topics. Simple experiments on wave motion, light, and sound that can be performed with a minimum of equipment are appended. (BB)

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EXPERIMENTS
ON
PHYSICS
IN
THE ARTS

1976

SE 025 114

THE UNIVERSITY OF SOUTH CAROLINA

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EXPERIMENTS

Staff Member - Ron Edge

Participants - Brock Dale, Bill Savage, Harvey Kaplan, George Bissinger

Experiment Number

1 . . . . H. K. The Vibrations of a Loaded String
2 . . . . B. D. The Vibration of a Stiff String
3 . . . . G. B. Vibrating Plates
4 . . . . H. K. Additional Experiments on the Properties of Vibrating Plates and Membranes
5 . . . . W. S. Vibrating Plates
6 . . . . W. S. Vibrating Air Columns
7 . . . . R. E. The Air Vibrations in a Trumpet
8 . . . . R. E. Oscillations in Cylindrical and Conical Pipes
9 . . . . R. E. Waves on Strings - The Guitar
10 . . . . R. E. Velocity of Sound
11 . . . . W. S. Room Acoustics - Reverberation
12 . . . . R. E. A Simple Anechoic Chamber
13 . . . . W. S. Transmission and Reflection of Sound
14 . . . . R. E. Experiments on Interference Between two Coherent Sources

Staff Member - LeConte Cathey

Participants - Frank Byrne, Bill Hartman, Dan Haines

Experiment Number

21 . . . . F. B. Vibrating Strings
22 . . . . F. B. A Student Perception of Vibrato
23 . . . . W. H. Loudness Levels
24 . . . . L. C. The Virtual Professor
25 . . . . W. H. Reverberation Spring
26 . . . . D. H. The Air Resonance of Guitars and Violins
27 . . . . W. H. The Resonances of Spoons

Staff Member - Richard Childers

Participants - Mary Taube, Terence McEnally, Ed Hall, Tom Brylawski

Experiment Number

31 . . . . M. T., E. H. The Mixing of Colors
32 . . . . T. M., M. T., E. H. Observations on the Spectra of Colored Lights and Objects
33 . . . . M. T., E. H., The Mixing of Colors
34 . . . . T. B. An Appendix on Color Mixing
35 . . . . M. T. The Camera and the Projector
36 . . . . T. B. Mirrors and Symmetry - Five Experiments
37 . . . . R. C. Adjacent Colors
38 . . . . R. C. Human Eye
39 . . . . Steve Shaffer - Shadows

Staff Member - Joe Johnson

Participants - Stanton Truxillo, Ed Watts, Tom Rossing

Experiment Number

41 . . . . S. T. Response of the Eye and the Ear to Intensity Changes
42 . . . . J. J. Scale Construction
43 . . . . E. W. Simple Harmonic Motion and Resonance
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Staff Member - Rudy Jones

Participants - Bill Hedgpeth, Danny Overcash, Linda Payne, Bill Skinner

Experiment Number

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Simple Experiments which can be performed with a Minimum of Equipment
Introduction

Wave Motion
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Light
A55. Reflection of Light
A56. Refractive index of water
A57. Optics--positive and negative lenses
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A59. Refraction of particles

Sound
A60. The rubber band guitar
A61. The effect on pitch of string tension
A62. Waves in pipes--wind instruments
A63. The Reed
A64. Combination tones
A65. Resonance in Sound
A66. The string telephone

INTRODUCTION

Many universities and colleges are now offering courses showing how physics relates to music and art. However, most laboratory experiments in the past have been designed for future physicists, and tend to "turn off" art and music students. It was our aim in putting together this manual, to design a series of experiments which would be stimulating and instructive to students of the arts, whose approach to experimentation is so different from science students.

The experiments in some cases, duplicate certain features, since they were devised by different groups. However, it is felt that in order to circulate this for use in the immediate future, it would be better not to alter them. The experiments are of very varied character and level, and it is hoped that different sections will appeal to different teachers.

In discussing an experimental course for musicians and artists, we must first consider what concepts we wish to convey. The "scientific approach" is probably the most important new idea to students whose primary motive is to influence our senses, although to put down in concrete terms what is meant by this approach is often difficult. The first consideration is probably to define the problem. In many cases it is more important to ask the right question than to find the right answer, since there is no advantage to answering a trivial question. In the first experiment, for example, one might ask, is there a relationship between the period of a spring and the mass it supports? Such a causal relationship was not obvious to the early scientists, and the somewhat mystical way in which Robert Hooke describes his experiments makes interesting reading today. Then we may ask, how can we study such a relationship? Physicists differ from observational scientists, such as geologists or botanists, in generally trying to perform experiments in a laboratory under controlled conditions,with the accompanying problems of accurate measurement. Thirdly comes the analysis of the results, and the confirming or refuting of a mathematical relationship between the quantities being observed, to some known degree of accuracy. The problem of accuracy, even if treated in a very elementary way, is of importance, since many physical laws (e.g. those of Newton) are obeyed only approximately. Lastly, we may ask, how do these results relate to other things, such as other properties of matter, and is there a more fundamental explanation for the results, fitting into the general pattern of physics - for example, in the case of Hooke's Law, this would be in terms of the atomic and molecular forces of a solid to the extent that they are linear or proportional to displacement to a first approximation.

In addition to the logical deduction fron observation, which is the basic mechanism physicists use to develop their science, we must ask what branches of physics are of most interest, or relevant, to musicians and artists. In addition, within such branches we wish to introduce concepts, or perhaps a way of looking at things, which would not otherwise occur to the student, but the way in which we introduce these concepts must be such that they become or bear directly on, the way of life of the student. The object is to use, as examples, things with which the student comes into constant contact, to reinforce what he has been told or found out. The problem with many elementary science courses is that the subjects of study are frequently not things with which the student normally comes into contact. He may see a Geiger counter once in his life, but the concept works the wrong way round. He says - gee whizz - a Geiger counter - that's the thing I did in the physics course. But what we would like him to say is - I must change the strings on my violin, they are non-uniform, and that produces poor harmonics, as I showed myself in physics. The point being, he will know why his violin sounds poor. What aspects of physics, then, are useful to musicians and artists?

To start with, we must examine the overlap between art, music, and physics. Their common meeting place lies in wave motion. Musicians are interested in sound, and artists in light - even sculptors look at the play of light on their statues. Hence, we must clearly emphasize experiments on sound and light.

Experimental physics courses conventionally start with an exercise on vectors, followed by experiments on velocity, momentum and energy. Time effectively rules out such experiments for music and art students, and it is better to insert such concepts into other experiments,using the relationship between energy and intensity for waves, both in the case of light and sound, and showing that power in watts of sound or light is the same as the wattage on a light bulb. We have found that it is necessary to devote the larger part of the course to these experiments on wave motion, light and sound, if students are to get a thorough understanding. A cursory survey proves useless since the students are unable to follow the new, and to them rather esoteric, ideas.

The initial experiments should be devoted to the general concept of vibrations - amplitude, frequency, and the nature and types of tone producing simple harmonic motion. There are advantages to using springs to provide the S.H.M. force, either a weight on a spring, or a glider on a linear air trough, since Hooke's Law can be discussed, which is later used to explain vibrations in musical instruments, reeds, gongs, strings and so forth. The extension of Hooke's Law to wave motion can be demonstrated by hooking together successive weights, or gliders, using springs. Such experiments are given in the text.

In connection with frequency and amplitude, it is useful at this stage that music students should be capable of associating their intuitive or trained concepts of pitch and loudness with the physical quantities Hertz and intensity. This can be done simply by clamping a piece of spring steel to the table so the vibration can easily be seen at low frequency, and the pitch heard to go up as the length vibrating is shortened. The amplitude of vibration is also clearly seen to be associated with loudness. A next step might be the idea of traveling waves. The concept of wavelength and velocity, and their relationship to frequency should be demonstrated. A suitable experiment for this is the speed of sound in air given in the text. This is conceptually simplier and aesthetically more appealing than the usual experiment involving a tuning fork and resonant water filled tube. Logically the next subject should be standing waves, leading to vibrations in musical instruments. Waves on strings are convenient to start with, since such waves are easily visualized. Standing waves on strings can be used to demonstrate modes of vibration, harmonics, and the relationship between tension, mass per unit length, speed, wavelength and frequency on a string. It is particularly interesting to use a non-uniform string to demonstrate anharmonic modes, since it is difficult for non-music students to realize the importance of using a uniform string to obtain a pleasant. harmonic tone.

Experiments on standing waves in pipes should follow those on strings, emphasis being placed on demonstrating the odd harmonics found in a closed pipe, and the full range of harmonic found in a pipe open at both ends or a conical pipe.

Having experimented on the one dimensional wave motion occuring in pipes and on strings, it seems logical to extend to two and three dimensions. The concept of interference provides us with a convenient method to introduce this. Two dimensional twin source interference can easily be demonstrated using a ripple tank, and then extended to two sound sources, such as loudspeakers or 40 kc transducers. The latter have the advantage and disadvantage of being inaudible, and the interference field is conveniently small. Then one can go to light sources, using a laser. This emphasizes the common qualities of wave motion. It also serves to introduce light as a wave motion, and further experiments on the wave nature of light follow naturally. Experiments on the velocity of light are rather complex and involved, and best omitted except for bright students. The relationship between color and wavelength is quite involved, because of an intuitive perceptual feel for color, and one or two periods are best spent to drive home what is meant by continuous and discreet spectra, additive color processes, the color triangle, and various optical illusions of importance. These prove interesting to the students, since it is something in their everyday life they were generally unaware of, and they tend to have a new regard for the analytical approach to everyday observations.

Optical instruments and ray optics is something students should be aware of. A knowledge of the working of the camera, telescopes and projectors, and the optical properties of the eye are particularly important to art students.

The relationship of light and sound to the environment should be studied. It is important to know noise levels in various situations, in the office, outside, near a car etc, and this can be conveniently measured with a portable sound level meter. Light levels are similarly measured with a foot candle meter. It is important for students in the media arts to understand the way in which illumination and exposure are connected for cameras, the product of speed and aperture being constant. Similarly, a knowledge of the meaning of dB on a sound level meter is something any recording artist will need. Both sound and light levels should be connected with the fundamental concepts of power and energy.

The experiments discussed so far are more than enough to fill one semester, and they proceed logically from one point to the next. However, we may well ask the question, what other experiments would interest, particularly, art students? Students of ceramics must be interested in rheology, since the special fluid behavior of clay is of importance. An experiment on elastic constants, and viscocity might be of interest, the effect of temperature etc.

THE VIBRATIONS OF A LOADED STRING

The vibrational properties of a uniform, flexible string with fixed ends form the cornerstone of a large part of what we call music, because the characteristic vibration frequencies of such a string, f_n, are integral multiples of the fundamental vibration frequency, f_1, of the string:

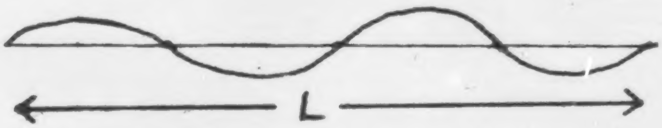
$$f_n = nf_1 \qquad n = 1, 2, \ldots$$

(The short way to say this is that the string's overtones are harmonics.)

It is also of interest to study the vibrational properties of a non-uniform string both for practical reasons, and also in order to deepen our understanding of the properties of vibrating systems. String players are all too aware that they may have to discard a string before it breaks because it becomes what violin players call "false". i.e. if two adjacent old strings, such as the d and a strings, are tuned to a perfect fifth and the two strings are stopped at positions which should be a fifth higher, at e and a respectively, this new interval of a fifth made by these notes will be out of tune if the strings are false. This difficulty can be attributed to a lack of uniformity of the string in either its mass density or its elastic properties. The problem is serious to the player because a set of new strings for a violin can cost over twenty dollars. The player often compromises by contorting his fingers when playing double stops, but it is a losing battle, better given up before being started. There is no solution for this problem, but it is perhaps of some interest to understand why the effect occurs.

A musical instrument problem which may be able to be solved with the help of the property of inhomogeneity in a string concerns the so-called octave stretching in the tuning of a piano. It has been established by test that the amount of octave stretching in a large grand piano is more pleasing than that in smaller pianos. It has been shown in a mathematical study that an appropriate small weight added near the end of a piano string can considerably reduce the octave stretching effect for that string. Although the idea has not been put into practice, and may even be impractical for thus far unexpected reasons, it is certainly of interest to understand the effect of inhomogeneity.

The experiment to be performed concerns the simplest possible inhomogeneity in a string property: A mass M approximately concentrated at a point is added at a point a distance l from the left (fixed) end and a distance l from the right (fixed) end of an otherwise uniform string of length L(l+l =L), density ρ and tension T. The effect will be exaggerated for emphasis and ease in measurement in that the mass M will be chosen to be about as large as the mass m of the entire string (m=ρL). The main experimental result will be a comparison of the normal mode frequencies of the loaded and unloaded strings. Before performing the measurements it is useful and possible to get a qualitative idea of the results to be expected. Consider a mode of the unloaded string as drawn



We expect that adding some mass to the string will decrease the frequency of any vibrational mode because such a frequency is inversely proportional to the square root of the mass undergoing vibration. If we want to lower the frequency of a particular mode as much as possible it is plausible that the mass be added at a point of the string that is vibrating most strongly in that mode. Any antinode of the mode is certainly such a point. If on the contrary we attach the mass at a node of a mode we might expect no change in that mode frequency! This argument has the consequence that if the mass is added at a fixed point the frequencies of the various modes will each be affected differently; for each of the above reasonings we might even conclude by experiment that the frequency changes are whimsical or random.

The theory of the vibrations of strings can predict the normal mode frequencies of our string in terms of the masses M and m, the length of the string L, the distances l+l , and the Tension T in the string. Because of the uneven way in which the various modes are affected, the equation for the frequency is implicit, meaning that the frequency appears in such funny ways in the equation that we cannot isolate it from all other quantities appearing in the equation. This is a common occurence in mathematics and no cause for alarm. It merely means that we will use a computer to determine the numerical values of the mode frequencies. Just for interest the equation to be solved for the frequency, f, is shown here.

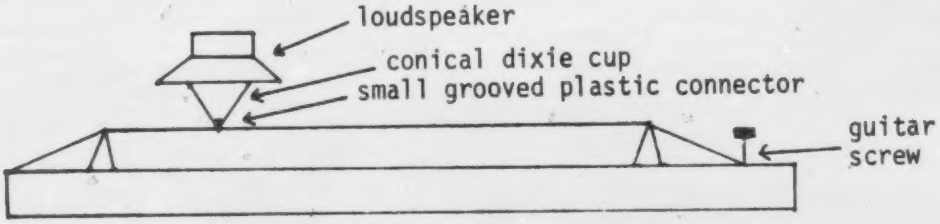
$$0=\left[\left(\frac{M}{m}\right) L2\pi\sqrt{\frac{m}{T}}\,f - \cot\left(2\pi\sqrt{\frac{m}{T}}\,\ell f\right) - \cot\left(2\pi\sqrt{\frac{m}{T}}\,\ell' f\right)\right] \qquad (1)$$
$$\times \sin\left(2\pi\sqrt{\frac{m}{T}}\,\ell f\right) \sin\left(2\pi\sqrt{\frac{m}{T}}\,\ell' f\right)$$

Equipment and Materials

1. A stretched piano wire of diameter 0.020" one meter long.
2. A piece of slotted lead shot used by fishermen, having a mass of between one and two grams.
3. A ruler
4. A balance
5. A loudspeaker driver to drive the string. The speaker is fed by an audio oscillator with a sine wave output with an intermediate amplifier,or transformer if needed and is mechanically coupled to the string near one end. A magnetic driver is also a possibility.

Procedure

1. The mass of the lead shot and the mass of the string are measured.
2. The string length is measured.
3. The frequencies of several modes of the unloaded string are measured by simply tuning the audio oscillator to a frequency that causes a maximum excitation for each mode. The tension of the string is established from the fundamental frequency. It is important that the mode frequencies be measured and not assumed to be harmonics of the fundamental frequency, because the piano wire is stiff and won't behave exactly like a flexible string.
4. The lead shot is secured to the string at various positions and modal frequencies are measured for several modes in each position. One position should be reasonably far from dividing the string into two lengths rationally related by small integers. Other positions used should place the mass M at l=1/2L, 1/3L, and 3/4L. It should be explicitly verified that the frequencies of modes with nodes at 1/2L, 1/3L and 3/4L are unchanged by the mass M.
5. The results of these frequency measurements should be compared with computed values from equation 1.



THE VIBRATION OF A STIFF STRING

The simple theory of vibration of a string assumes that it is infinitely flexible; that is, that a free string could be bent and held to any angle without exerting any force on it. This of course is not true for any real string because all materials have some stiffness. Part of the other force that impels a string to move toward its original position after it is pushed or struck is due to its stiffness. Therefore, the vibration of real strings deviates in some way from the vibration of ideal flexible strings, and one may expect that as the stiffness of the string increases, its vibrational frequencies will deviate more and more from those of an ideal flexible string.

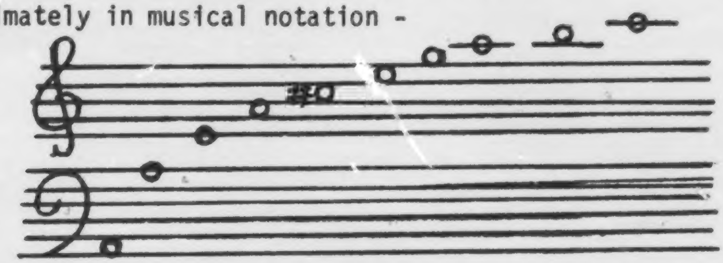
In an earlier experiment* you have seen that for a flexible string the frequency of the string when vibrating in its $n^{\underline{th}}$ mode (with n antinodes between the ends of the string) is

$$f_n = \frac{2n}{\ell}\sqrt{T/\mu}$$

where T is the tension in newtons, μ is the mass of the string per unit length in Kg/m (a rather small number), n is the number of the mode or harmonic being excited, and ℓ is the length of the part of the string that is vibrating.

So that $f_2 = 2f_1, f_3 = 3f_1, f_4 = 4f_1$, etc.

This gives rise to the familiar harmonic sequence which can be written approximately in musical notation -



Suppose we consider, instead of a flexible string under tension, a round elastic rod with its ends clamped, but without tension. The fundamental frequency f_1 is given by

$$f_1 = \frac{3.5608}{\ell^2}\sqrt{\frac{\text{stiffness factor}}{\text{mass per unit length}}}$$

where ℓ is the length of rod between the clamped ends. The overtones are not integral multiples of the fundamental. Rather, we have the rather odd set of overtones -

$$f_2 = 2.7565f_1$$

$$f_3 = 5.4039f_1$$

$$f_4 = 8.9330f_1$$

$$\vdots$$

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*Experiment No. 11

The velocity of a transverse wave in a rod is not independent of the frequency, so one must write $f_n\lambda_n = V_n$, the subscript n on the velocity V indicating that V is different for the different allowed frequencies f_n.

The stiffness factor in the above equation is given by $Q\pi r^4/4$. Q is a property of the material from which the rod is made, called its Young's Modulus of Elasticity and r is the radius of the rod.

As might be expected, the vibration of a stiff string, depending as it does on both the stiffness and the tension, is a complicated expression involving the stiffness, the length, the mass per unit length and the tension and all of these factors must be specified before f_1 and the ratios f_2/f_1, f_3/f_1, etc. can be specified. However, a rather simple approximation is valid as long as the ratio of the stiffness to the tension is not too large. To this approximation, the fundamental frequency f_1 is

$$f_1 = f_{1\ flex}\left(1 + \frac{2}{\ell}\sqrt{\frac{\text{stiffness}}{\text{tension}}}\right)$$

where f_1 is the frequency at which the stiff string vibrates in its fundamental mode when it has a certain length and tension, and $f_{1\ flex}$ is the fundamental frequency of a perfectly flexible string having the same mass, length, and tension. We shall call the factor -

$$\frac{2}{\ell}\sqrt{\frac{\text{stiffness}}{\text{tension}}}$$

the Stiffness Correction (abbreviated S.C.).

To this approximation the harmonics are integral multiples of the fundamental, i.e., $f_2=2f_1$, $f_1=3f_1$, . . . $f_n=nf_1$. This approximation is sufficient to account partly for the fact that fretted instruments, such as the guitar, mandolin, or banjo may be in tune in so far as the open strings are concerned, but go progressively more out of tune as they are played at higher and higher frets on the fingerboard (i.e., as the fingers are placed down nearer and nearer the bridge).

First, note that $\frac{1}{2\ell}\sqrt{\frac{\text{stiffness}}{\text{tension}}}$ increases either as ℓ or the tension decreases, or as the stiffness increases. Therefore, this factor, which one may call the stiffness correction, increases as the string is made shorter or less taut, and for a given length and tension, it will be larger for stiffer strings than for less stiff ones. Let us consider how this factor affects the intonation of a rather stiff string under low tension.

The open A string of a guitar is normally tuned to 110 Hz. If the string is made to contact the twelfth fret, it should vibrate at 220 Hz, an octave above the open string. Suppose this fret were exactly at the middle of the string, as it should be for a perfectly flexible string. If the open string is tuned to 110 Hz, then

$$f_1 = f_{1\ \text{flex}}\ (1 + \text{S.C.}) = 110\ \text{Hz}.$$

where S.C. is the Stiffness Correction

Now suppose the string is pressed against the twelfth fret. The quantity $F_{1\ \text{flex}}$, which, it will be remembered, is $\frac{1}{2\ell}\sqrt{\frac{\text{tension}}{\text{mass per unit length}}}$ is doubled, because ℓ is only half as big as it was. The new f_1 is $f_{1\ (\text{half})} = 2\ f_{1\ \text{flex (open)}}\ (1 + \text{new S.C.})$

The new stiffness correction is different from the old one because of the presence of ℓ in the denominator. When ℓ is reduced to half, the S.C. is doubled. If it remained the same, the new frequency $f_{1\ (\text{half})}$ would be just twice the old one and the sound would therefore be exactly one octave above the open string. But since the stiffness correction has doubled, the new frequency is in fact too high by an amount -

$$2f_{1\ \text{flex}}\ (\tfrac{2}{\ell})\sqrt{\text{stiffness/tension}}$$

i.e. by $2\ f_{1\ \text{flex}}$ (old S.C.). This is, to a good approximation just (220 Hz) (old S.C.), and the ratio of the new frequency to the true octave is just $\frac{220 + 220\ (\text{old S.C.})}{220}$ = 1 + old S.C., or to recover our mathematics

$$f_{\text{half}}/\ f_{\text{octave}} = 1 + (2/\ell_{\text{open}})\sqrt{\text{stiffness factor/tension}}$$

On a properly constructed steel-string guitar this effect is compensated by sitting the bridge at an angle so that the fret-to-bridge spacing is a little larger for the heavy strings than for the light ones.

However, on a twelve string guitar, some adjacent strings are tuned in octaves, and are of different stiffness. If such strings are consonant when unfretted, they are out of tune and beat when held to the lower frets because of the stiffness effect. This is a serious problem for the twelve string guitarist.

In the experiment outlined here, you will determine the effect of the stiffness of the string by comparing the frequency of a heavy string stopped exactly at its center with that of a light string similarly stopped.

Apparatus:

1. Guitar with .010" diameter string tuned to E_4 and a .030" diameter string tuned to E_2, with a fret placed at exactly the center of these strings.*
2. Small magnetron magnet mounted on a frame so it can be positioned near the string.
3. Sawtooth oscillator (or adjustable-width rectangular wave oscillator), amplifier and speaker.
4. Sine wave oscillator, amplifier, transformer, 15 ohm to 1 ohm, 1 ohm 5w resistor.
5. Clamp for fastening strings against the octave fret.

PROCEDURE

General:

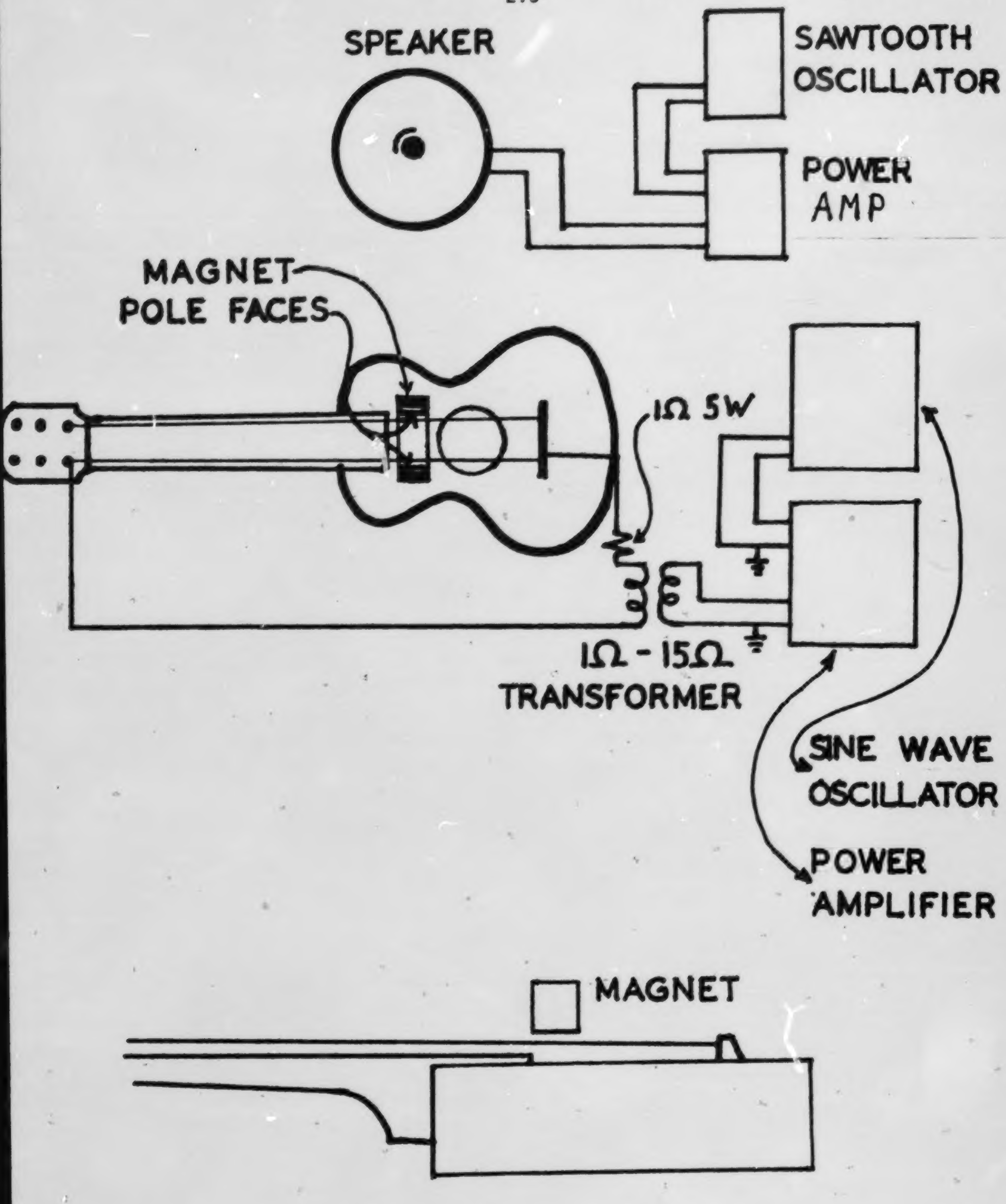
The general procedure is to set the open string into vibration by passing an alternating current through it at its fundamental frequency. Since it is in a magnetic field an electromagnetic force is imposed on it that varies at the frequency of the applied alternating current & this sets the string into oscillation. The frequency of the fundamental is matched exactly with the fundamental of a sawtooth wave generated by a function generator. The sawtooth is left at the same frequency, and the string is divided exactly in half by clamping against the "center" fret, and the new frequency is compared with the second harmonic of the sawtooth. Since this second harmonic is exactly one octave above its fundamental, it is also exactly an octave above the frequency of the open string. The difference between this frequency and the frequency of the clamped string is just the amount by which the frequency of the divided string is in error. This difference is determined by counting the number of beats per second when the half-string and the sawtooth are sounded together. This number of beats per second is exactly equal to the difference in frequencies, i.e., it is the error in the frequency of the half-string.

Specific Procedure:

1. Read all the foregoing carefully. Don't connect anything to the wall socket until your circuit has been checked by the instructor.
2. Tune the heavy string on the guitar to E_2 (about 82 Hz) and the light string to E_4 (about 330 Hz).
3. Lay the guitar on its back on the lab table and put the neck rest under the the neck near the end.

* The fret should be a little above the level of the normal frets on the guitar, but not touching the strings.

2.5


SPEAKER
SAWTOOTH OSCILLATOR
POWER AMP
MAGNET
POLE FACES
1Ω 5W
1Ω - 15Ω TRANSFORMER
SINE WAVE OSCILLATOR
POWER AMPLIFIER
MAGNET

4. Position the magnet above the heavy string, as close as possible without touching it. The pole faces should straddle the string.

5. Connect one output lead from the transformer to one end of the string (past the nut) and the other output lead to the resister. Connect the other end of of the resister to the other end of the string between the string slot and the bridge.

6. Connect the input leads of the transformer to the speaker output of one of the amplifiers.

7. Connect the output from the sine-wave oscillator to the tuner input of the amplifier.

8. Connect the output of the sawtooth oscillator to the tuner input of the other amplifier.

9. Connect the speaker output of the amplifier to the speaker.

10. Set both oscillators to about 80 Hertz.

11. Have all circuits, settings and mountings checked by your instructor, then plug the instruments into power sockets and turn them on.

12. Tune the sine wave oscillator carefully over a 10-15 hertz range, and watch for oscillations of the string. When these begin, tune very carefully to get the maximum amplitude of vibration of the string. Leave the sine wave oscillator at this setting. If the string buzzes against the fret, turn down the amplifier volume control and carefully re-adjust the tuning.

13. Tune the sawtooth oscillator to the same frequency. Dials may be a little in error. As the frequency of the tone from the speaker approaches that of the guitar string, regular changes in loudness will be heard. These are beats. They will become slower as the two frequencies get closer together. Tune to get less than one beat in five seconds.

14. Leave the sawtooth oscillator frequency dial where it is, but turn down the amplifier volume. Turn off the amplifier driving the string. Move the magnet to about halfway between the center of the string and the bridge. Clamp the string against the fret.

15. Turn the amplifier back on, adjust the frequency to about 160 Hz, and tune again for maximum amplitude of vibration of the string.

16. Turn up the volume control on the other amplifier and count the number of beats per second. Record this number.
17. Turn both amplifiers off, re-position the magnet over the lighter string.
18. Repeat parts 12-19, using frequencies of about 330 and 660 Hz.
19. If time permits, replace the light string by a wound guitar E string, tune to E_2 and repeat 12-19.

Questions:

1. You have probably found that when you used the light string the frequency of the half-string was not measurably different from twice the frequency of the open string. Why?
2. The length of the upper half string can be increased to lower its pitch, and $f = 1/2\ell \sqrt{T/\mu}$. The change in frequency Δf is given by:

$$\frac{\Delta f}{f} = \frac{\Delta \ell}{\ell} \qquad \text{or} \qquad \Delta \ell = - \ell \frac{\Delta f}{f}$$

Δf is the frequency change, f is the original frequency $\Delta \ell$ is the increase in length, ℓ is the original length. If $f = 220$ Hz., $\ell = .63$ m, how much would you have to move the bridge away from the fret to make the pitch correct? Use your own value of Δf.

3. Does the slant of the bridge account for this $\Delta \ell$?
4. What other effect might make it reasonable to make the heavy strings longer than the light ones? (Hint: What happens to the tension when you press the string against a fret?)
5. What is the advantage of using a wound string like the regular guitar E string?

Vibrating Plates

Any discussion of the acoustical properties of the string instruments must finally arrive at the vibrations - not of the strings but of the wood plates which form the top and bottom of these instruments. This is due to fact that although the musicians bow or pluck the strings, you hear only the vibrations of these plates. Traditionally, Luthiers have always "tuned" these parts of the instrument by ear but recent work using techniques such as holography,[1,2,3,4] ,resonance analysis,[5,6] on violins has shown that the presence of a well defined and prominent "ring" mode is common to all the good violins (Strads, etc.) tested. This mode, where the entire surface of the plates (top and bottom) is in motion (except for a narrow band) contributes in a major way to the full bodied, clear tone of a good string instrument.

This experiment deals with the vibrations of square or rectangular plates (which have fundamental frequencies which it is possible to calculate theoretically) and those of violin shaped plates, both flat and arched as in actual violins, which it is not now (or maybe ever) possible to calculate theoretically.

Equipment:

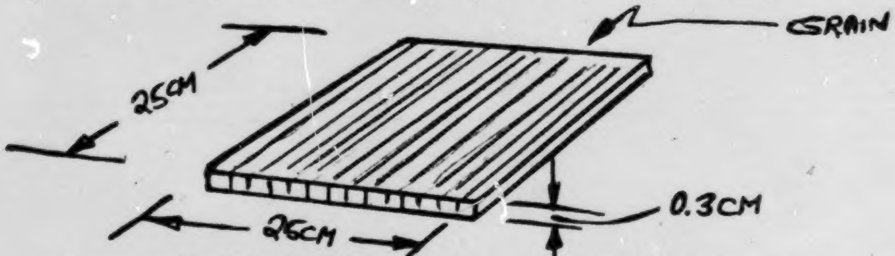
1) An 8" loudspeaker capable of covering the 50-1000 Hz region with powerful output. It is necessary that this be a speaker of reasonable quality without a substantial amount of harmonic distortion (< 5% at 50 Hz) A power rating of 10-20 watts continuous is recommended.

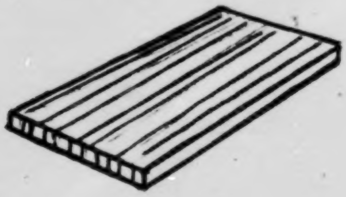
2) A good quality 20 watt RMS Amplifier (mono is OK) (at least over frequency region 50-5000 Hz).

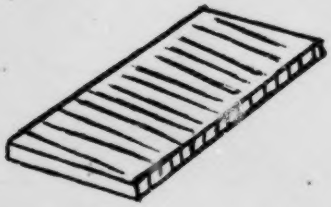
3) A sine wave generator - either continuously variable or with

frequency steps no larger than 1 Hz over 50-1000 Hz. It should be capable of driving the amplifier directly.

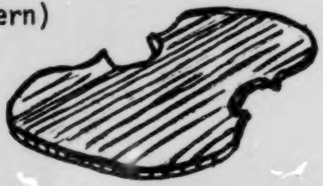
4) 2 spruce or pine plates (1 square or 2 rectangular) with dimensions of the order of 25cm square or 25cm x 35 cm. The plates should be 0.3cm thick and of uniform thickness. NOTE: grain in the plates should run parallel to one side, and perpendicular to flat surface. If the plates are rectangular the grain should run parallel to the long dimension for one sample and perpendicular to the long dimension for the other.







5) Flat spruce or pine plate cut in shape of violin (see enclosed pattern) The grain must run parallel to the center of the plate, and the thickness should be about .3cm.



6) One violin top or bottom plate (can be gotten either from cheap violin - make sure the person taking apart violin is skilled - or by forming it yourself

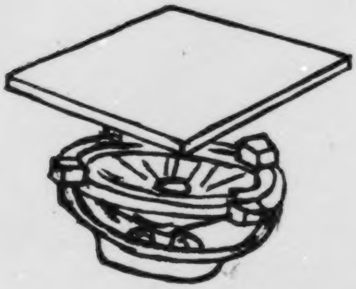
7) "Glitter" - The aluminized plastic speckles available in stores, etc. usually come in salt shaker container.

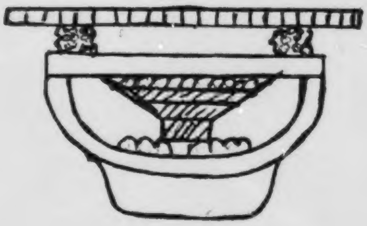
8) Foam rubber blocks (four) about 1/4"-3/8" thick by 1/2"x1" in cross section (foam should be as porous as possible).

9) Protective ear coverings. The sound levels necessary for this experiment can get uncomfortably high. (Try to set up experiment in a room that is "dead" acoustically). A pair of earphone-like coverings work well, or even the isolating type regular earphones.

I. Vibrations of a rectangular (square) plate.

a) Place the plate directly over the loudspeaker using the foam blocks (which can be attached to loudspeaker support with two sided tape).





For best results the plate should be close to the loudspeaker but never touching any part of the loudspeaker.

b) Sprinkle "glitter" evenly over surface of plate.

c) Adjust signal generator to 30Hz and turn up signal level till sound is clearly audible but not loud. (Do not drive speaker into harmonic distortion).

d) Then sweep frequency higher (slowly!) until "glitter" starts to move around.

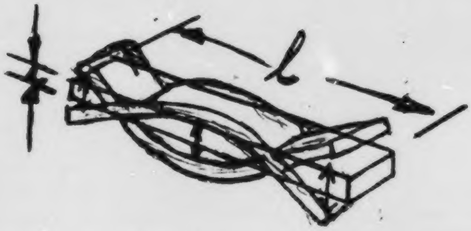
e) Tune generator until "glitter" seems most active.

f) Turn up signal level until you can clearly see "glitter" bouncing off surface and adjust frequency if necessary to get sharpest pattern.

Discussion of I.

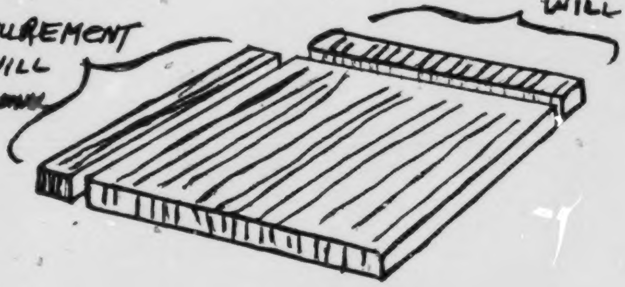
The fundamental frequencies of vibration of a rectangular plate free to vibrate in all places can be gotten from a relationship derived for a rectangular bar of the same material,[8] free to vibrate in all places (eg. 1)

(1) $$f_o = 1.028 \quad \frac{Y}{\rho} \quad (\frac{a}{\ell^2})$$



a = thickness
ℓ = length
Y = Youngs Modulus
ρ = Density of material

NOTE: When the wooden plate(s) is/are prepared it is useful to have pieces of the plate for testing purposes. The easiest way is to save the edges from the plate e.g.

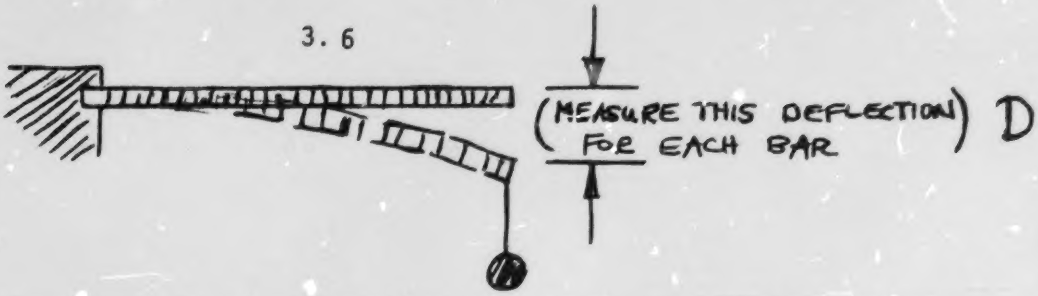


DEFLECTION MEASUREMENT ON THIS BAR WILL GIVE D_R (RADIAL GRAIN BAR)

DEFLECTION MEASUREMENT ON THIS BAR WILL GIVE D_L (LONGITUDINAL GRAIN BAR)

These two pieces should have exactly the same width (1/2"-3/4") and length (which should be close to length of plate). You will notice immediately that these two pieces have quite different stiffnesses, and this is what causes the two separate frequencies of vibration observed in I. It is a simple matter to find the approximate ratio of frequencies for these two fundamental modes of vibration by clamping both bars at one end and hanging a small weight from each in turn and noting the deflection of the end produced with the small additional weight.

3. 6



Start with floppy bar and put on a small weight that does not deflect this bar through more than 2.5 cm.

The ratio of these deflections can be related to the ratio of frequencies using

(2) $$\frac{f_L}{f_R} \approx \sqrt{\frac{D_R}{D_L}}$$

D_R IS DEFLECTION FOR RADIAL GRAINED BAR

D_L IS DEFLECTION FOR LONGITUDINALLY GRAINED BAR

For a medium like wood which has a variable stiffness and mass depending on grain direction, humidity, grain regularity - it is quite difficult under the best of circumstances to make accurate, reproducible measurements of masses, dimensions and stiffnesses. Hence the above ratio will not be very accurate (expect 25% error).

The mass of the plates and the volume must also be measured. Typical densities (ρ) are .4-5 g/cm^3. It is best to use a measured value of ρ due to humidity, local variations, etc.

The values of Y (Youngs Modulus) are usually measured for wood by using the equation for frequency, measuring the fundamental frequency and then calculating what Y is. Measurements of this type give values for Y for bars bent against the grain (Y_L) and with the grain (Y_R). Values for a sample of Vermont spruce are

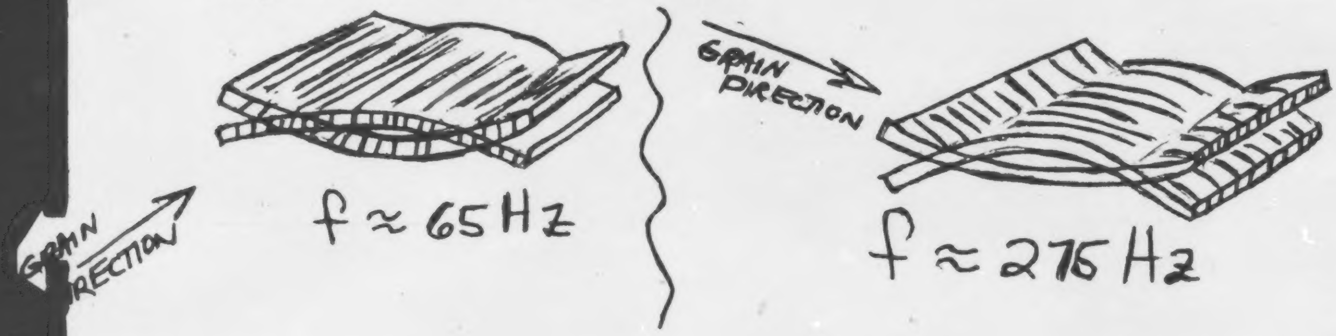
Y_L = 13 x 10^{10} dynes/cm^2 (stiff bar)

Y_R = 7.2 x 10^9 dynes/cm (floppy bar)

The variation in measurements of Y_L is usually much less than in Y_R. The velocity of sound in the material is proportional to the square root of Young's modulus, so sound travels four times faster along than across the grain.

NOTE: Young's Modulus, Y, is an empirical factor that relates how much a bar contracts (or lengthens) when a pressure change is used to compress or expand the bar. e.g. A fairly soft rubber rod will have a relatively low value of Y while a hard steel bar will have a relatively high value of Y. It is difficult to directly measure Y for soft woods since they will compress or squash if too much pressure is exerted.

To calculate the fundamental frequency of vibration for the plate, it is probably most accurate to calculate the case for the "stiff" direction of vibration using the value $Y_L = 13 \times 10^{10}$ dynes/cm^2. Then the fundamental frequency of vibration in the "floppy" direction can be gotten using equation 2. Approximate values for vibration in the "floppy" direction can be gotten by using the value of Y_R given above. For a Vermont spruce plate with $\rho = .4$ g/cm^3, thickness of .3cm and 25cm square we get the two fundamental frequencies at $f \approx 65$Hz and $f \approx 275$Hz.

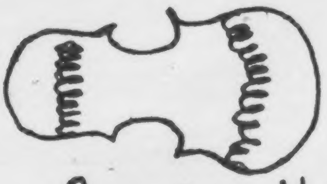


II. Violin shaped plates

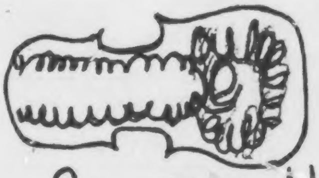
a) Place the flat violin shaped plate so that it rests on the foam blocks over the loudspeaker.

b) Set the oscillator at 100 Hz and slowly increase frequency until you get the lowest frequency glitter pattern that is associated with vibrations of the whole plate. It is not enough for just a small area of the plate to oscillate. At resonance(which ever one it is)you expect over 1/2 of the plate surface to be in motion.

If you reach 400 Hz without getting a sharp pattern go back to 100 Hz and turn up loudspeaker volume - If speaker starts "honking" you are driving it into distortion and must reduce volume until tone is again clean. Glitter patterns (Chladni patterns) observed for .3cm spruce plate looked like



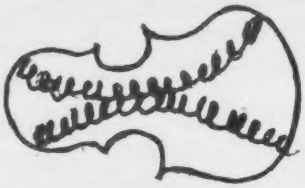
$f \approx 220$ Hz

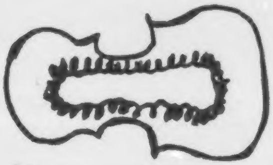


$f \approx 350$ Hz

You should get similar patterns at frequencies in the same region. The different shape of these violin shaped spruce (or pine) plates will give patterns entirely different from square or rectangular shapes.

c) Now set the violin top (or bottom) plate on the speaker and starting from 100 Hz, sweep the frequency up to 500Hz. Make a sketch of every sharp pattern in this frequency range and make sure you label each sketch with the frequency. The two lowest frequency patterns should look similar to





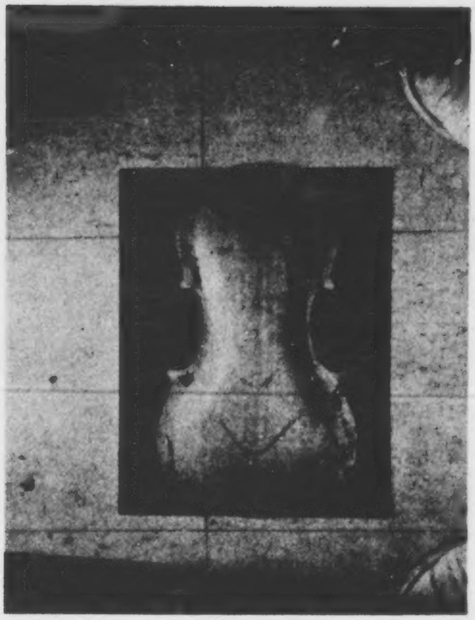
"RING" MODE

The pattern at the right corresponds closely to the "tap" tone that violin makers listen to when they tap the violin plates during the tuning (by ear) process. It is often called the "ring" mode.

d) Pick up the violin top (or bottom) and suspend between thumb and forefinger on a nodal line very lightly (but do not drop).

Then with 3rd or 4th finger tap the center of plate lightly and listen to the tone. You will probably have to put your ear quite close to plate to hear the tone. Try matching the frequency of the tap tone you hear with the signal generator. It should be close to the frequency you got for the "ring" mode.

e) It is interesting to fool around with the frequency generator and look at glitter patterns at the higher frequencies not only for the violin plates but also for the square or rectangular plates. It is possible to achieve extraordinary patterns for these vibrating plates. A nice additional demonstration of vibration patterns is possible for thin (1/16") brass or steel square or rectangular sheets (or circular).



Questions

1) How close did your measured square or rectangular wooden plate fundamental frequencies come to calculated values? Make a small table of experimental, and calculated frequencies and percent errors for each case.

2) Was the ratio of measured fundamental frequencies close to the values that you calculated from measured deflections? What was your percentage error?

3) Using the same sound levels for both the violin shaped flat and violin top plates, which was the most "active" at the two lowest resonances? You can tell this either by touching surface with the finger and determining which vibrated the most or by watching the height to which the glitter is kicked.

4) What difference between the violin top (or bottom) plate and the violin-shaped flat plate do you think causes the violin top to possess a "ring" mode while the flat plate does not? (This occurs even if both plates are the same thickness).

5) Describe the motion of the surface of the rectangular or square plate when it vibrates at a fundamental frequency (in either direction relative to the grain).

6) Describe as accurately as you can the motion the violin top (or bottom) plate surface undergoes when it vibrates in the "ring" mode.

3.11

REFERENCES

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2. E. Jansson, N. E. Molin and H. Sundin, Physica Scripta, 2, 243 (1970).
3. C. H. Agren and K. A. Stetson, J. Acoust. Soc. Amer. 46, 120 (1969).
4. W. Reinicke and L. Cremer, ibid., 48, 988 (1970).
5. C. M. Hutchins, A. S. Hopping and F. A. Saunders, ibid., 35, 1443 (1960).
6. C. M. Hutchins, Sci. Amer., Nov. 1962, P. 78-93. Phys. Today, 21, No. 7 (1968).
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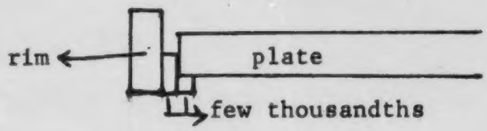
ADDITIONAL EXPERIMENTS ON THE PROPERTIES OF VIBRATING PLATES AND MEMBRANES

The apparatus used in the above experiments can also be used to compare the patterns and frequencies of the normal modes of geometrically identical plates and membranes. Because a membrane is elastically isotropic (i.e. equally hard to bend in any direction) and must have a fixed as compared to a free border, the plate with which it is to be compared should also be elastically isotropic and have either a clamped or a supported border.

The clamped plate consists of a uniform undented flat piece of metal clamped between two identically shaped (either square or round) heavy metal rims.

A supported plate is held by a rim on its circumference but is not clamped. The region of support should be as small as possible, as shown in the figure



The membrane consists of a thin uniform rubber sheet stretched to a desired tension between a pair of rims identical to those used for clamping the metal plate. The establishment of uniform tension in the membrane is accomplished by painting a square on the unstretched rubber, and stretching the rubber by successive approximations until the painted square is again square under tension. Unless a student has a lot of time the mounting should not be part of his experimental procedure. (This limitation is unfortunate in that one can learn a great deal about the nature of real things and their challenges and frustrations by stretching a membrane to reasonably uniform tension).

The procedures for both the membrane and the plate are the same. The object to be vibrated is placed over the hole in the vibration testing table in such a way that the hole through which the sound is transmitted from the speaker to the membrane or plate is not directly over a nodal line of the mode that is being studied. (The heavy rim should be separated from the table by thin foam gasket to prevent spurious vibrations). A number of modal frequencies should be determined and patterns of the nodal lines for each of the identified modes should be sketched, with linear dimensions measured when the shapes are simple, such as is the case e.g. for a linear mode. The student should also record the frequencies of the modes.

The fun of this experiment is then the attempt to relate the experimental results to a few simple results of the theory of vibrating membranes and plates:

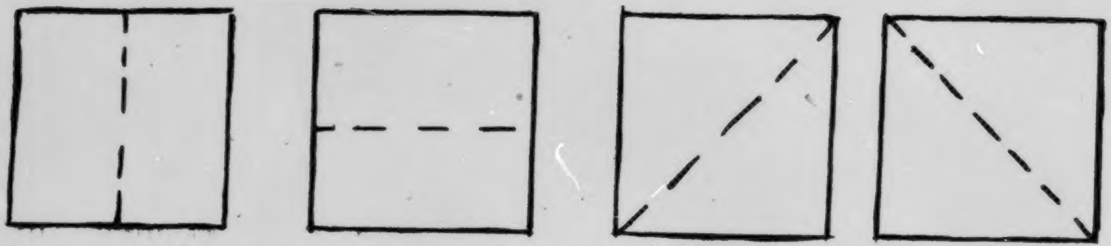
(1) The normal mode frequencies of a vibrating square membrane of side a are given by -

$$f_{mn} = \frac{1}{2a}\sqrt{\frac{T}{\sigma}}\sqrt{m^2 + n^2}\,, \qquad \begin{array}{l} n = 1, 2, 3, \ldots \\ m = 1, 2, 3, \ldots \end{array}$$

where T is the tension in the membrane, and σ is its mass per unit area. It would be hard to measure the tension T and so a better scheme consists in comparing all frequencies to the lowest frequency f_{11}:

$$\frac{f_{mn}}{f_{11}} = \sqrt{\frac{m^2+n^2}{2}}$$

The student's attention should be drawn to the fact that when m and n are unequal two modes exist with the same frequency. Of course it is impossible to create a perfectly symmetric experimental arrangement, and so any attempt to drive the membrane will stabilize one or another of the possible modes with its particular nodes. The student should be encouraged to try to stabilize at least two possible patterns for a given frequency f_{mn} with $m \neq n$. This can perhaps be done by placing a finger at a point of one nodal line of a desired pattern. As an example, the pair of frequencies f_{21} and f_{12} show the following 4 nodal patterns for a square membrane -



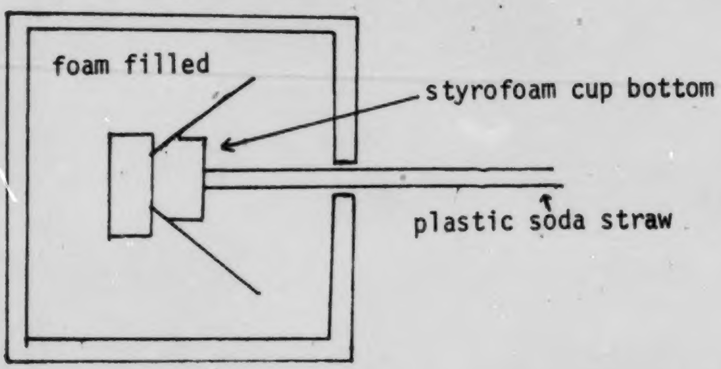
The square plate, either clamped or supported, is complicated to study mathematically but there is some point in expecting mode frequencies referred to the fundamental frequency to be reasonably close to -

$$\frac{f_{mn}}{f_{11}} \simeq \sqrt{\frac{m^2+n^2}{2}}$$

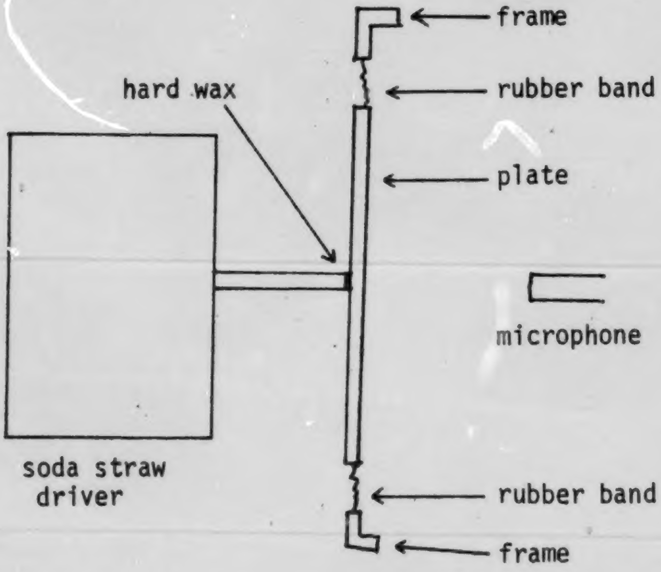
The main source of information about plate and membrane vibrations is the vibrant "Theory of Sound" by Lord Rayleigh, and in particular, chapters 9 and 10 in volume 1. The student may consult the few remarks made by Backus on the subject, and also his instructor.

VIBRATING PLATES

As a check of the vibrational frequencies obtained with the acoustically coupled speaker drive units,one can couple the vibrations directly. A coil can be mounted on the plate and a magnet clamped on a support brought near the coil so the magnetic field intercepts the drums of the coil. An improvised system can be made from an enclosed speaker. A plastic cup and soda straw can be glued (use bathtub caulk,which is air curing rubber) to the speaker cone. The straw is the only part that extends from the heavy wall wood box surrounding the coil.

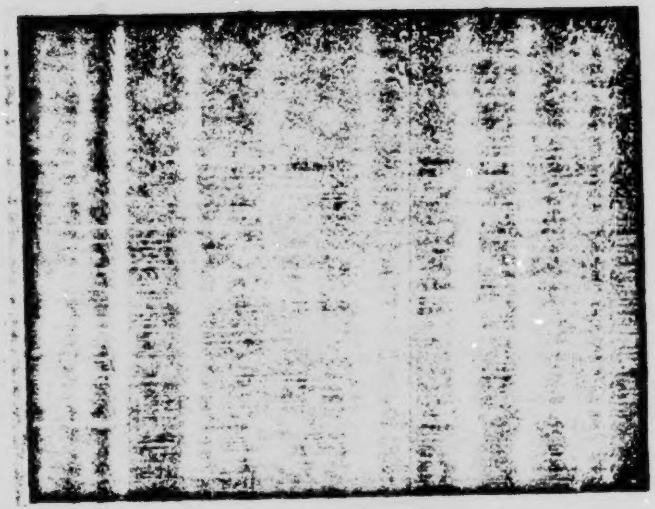

foam filled
styrofoam cup bottom
plastic soda straw

The plate to be tested is mounted by a rubberband support with the plate in the vertical direction. A 1/4 inch rod, which can be force fit to the straw, is waxed with hard sealing wax to the plate


frame
rubber band
hard wax
plate
microphone
soda straw driver
rubber band
frame

5.2

The driver is connected to an amplifier and frequency generator. The microphone is connected (with amplifier if needed) to an oscilloscope. As the frequency generator is adjusted to give higher frequencies large amplitude signals can be detected in the microphone at the resonant frequencies of the plate. These can be compared to the "tap-tones" of the plate or the acoustically coupled resonant frequencies. Account for any differences in the spectral response.



Vibrating Air Columns

Objective: Air columns can be set into resonance by a number of ways. One of the ways is to use the lip reed. A series of special air columns are to be set into resonance and the frequency measured. The fundamental of the tones produced is to be computed. The frequencies are to be compared to an ideal air column.

1. Straight Pipe: A piece of pipe has been fitted with protective ring at one end. Practice blowing until 3 or more steady tones can be produced. Measure the frequency by comparing the blown tone with a pure tone by the beat frequencies method. If a frequency counter is available it can be used to measure the frequency of the tones directly. A microphone and amplifier can be connected to the counter and the frequency read from the output.

   For the frequencies you measure, find the greatest common multiple and the integer multiplier for the tones produced. Compare these frequencies to that of ideal pipe resonances. Confirm these values for the actual pipe length. Apply an end correction and a reasonable value for the speed of sound for the temperature of air blown through the pipe. How does the lip-reed act acoustically? Compare the sequence of tones to a chromatic or equally-tempered scale. Is the pipe "in tune?"

2. Conic Pipe: Measure the frequency of tones produced from a conical pipe. In this case significantly more than 3 tones can be produced. Find the greatest common multiple and the integer values of the tones produced. Compare these tones to an ideal straight pipe of the same length. What is the effect of the conic cross section or "bore" of the pipe? Compare the sequence of tones to a chromatic or equally tempered scale. Is the conical pipe in tune?

3. Post Horn: Measure the frequencies for tones produced by a post horn. Compare these tones to those calculated for a straight pipe of the same length. These tones should be in close agreement to a chromatic scale even when produced by a beginner. Fit the straight pipe and the conical pipe with a mouth piece. Now compare the tone produced. Can the straight or conical pipe now be blown in a chromatic scale? The interior shape of the horn is complex. What effect has the flare or bell had upon the frequency of the tones?

4. Real Horn: a. Measure the length of some instrument such as a cornet or trumpet by laying a piece of string along the up-valve path. Without depressing the keys measure the frequency of the tone produced. Compare the results with part C.

   b. A mouth piece can be fitted with a microphone sensitive to pressure changes. The resonant frequencies can be found for the mouth piece alone by using an oscilloscope and Lissajou patterns. Compare these to the frequencies obtained for an instrument.

   c. A sub-miniature pressure microphone can be placed on a slender rod and used to probe the interior of the tubes or instrument down the first bend.

THE AIR VIBRATIONS IN A TRUMPET

Object: To investigate the working of a trumpet by examining the air oscillations in the mouthpiece, backbore, and bell.

Method: A special mouthpiece is required constructed as shown in figure 1. Very fine holes (less than 0.5 mm diameter) are drilled into the cup, and about half way down the backbore. A receptacle for a small microphone is attached over the holes with epoxy resin. One hole is closed with a little wax, and the microphone fits tightly over the other, so no air can leak around it. Even small leaks will alter the frequency, and make the instrument difficult to play. A microphone element, such as is used for the more expensive cassette recorders, proved very successful for this part of the experiment when removed from its usual container. The microphone is connected directly to an oscilloscope, no amplifier being necessary because of the very large pressure excursions. The trumpet is blown, and the oscillations photographed from the face of the oscilloscope using an internally triggered linear time base of, say, a millisecond per centimeter. Both cup and backbore should be photographed for middle C, the C above this and the high G above this. It is possible to get at least two traces on each film by displacing the oscilloscope beam in the y direction between exposures. The microphone element is then replaced in its normal container (this improves its frequency response) and placed in front of the bell of the instrument. It is interesting to note how the oscillations change for the same note, as the microphone is moved around outside the bell, showing different observers hear a different timbre for the same instrument and note. However, for our purposes, these effects are small compared to the overall wave shape. Again, the oscillations should be photographed for the same three pitches.

A very small microphone, (Figure 2), is now attached to the oscilloscope. Such "tie clip" microphones can be obtained for a few dollars. The pin attachment at the back is removed, but the microphone enclosure itself should not be opened, or the metal foil diaphram inside will be ruptured. The microphone is attached to a long thin support, such as a coat hanger wire, and the oscillations observed on the oscilloscope as it is inserted into the bell and then the straight section of the trumpet. It will be seen that phase oscillations or motion of the high frequency components occur, and the pattern changes very rapidly as a function of the distance from the end.

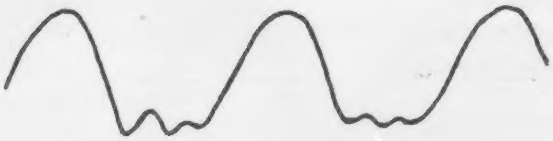
Results: The results consist of a sequence of pictures of the oscillations.

These can be treated in either a qualitative or quantitative fashion.

Qualitative:

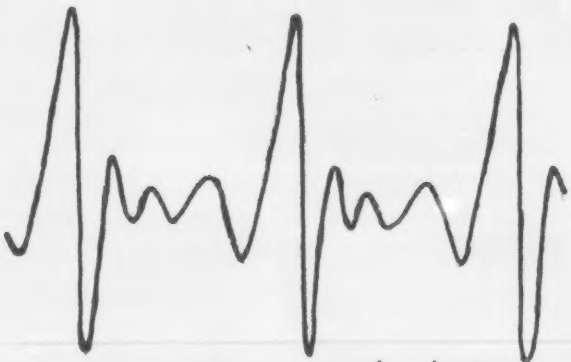
The oscillations measured in the backbore represent the pressure fluctuations at the closed end of the (more or less) uniform pipe forming the trumpet. Since the microphone is attached by an air tight seal to the backbore, it measures pressure variations and not particle speeds. All the normal modes of oscillations of a closed pipe have pressure antinodes at the closed end, and hence the picture of oscillations here should be the best measure of all the modes excited in the trumpet. Wavelengths shorter than the distance from the hole to the throat of the mouthpiece, will, of course, not be measured accurately. If we compare this picture with what comes out of the trumpet bell, we see that the fundamental is much stronger within the tube than outside. This is because the relatively small bell of the trumpet (4 1/2" diam) radiates the lower frequencies much less efficiently than the higher. This has the effect of what physicists and electronics engineers call "differentiating" the sound pulse. In the tube, The pulse looks like (a) in figure 1. After "differentiating" by the bell it becomes much sharper, and

(a)



C above middle C (C_5, 523.25 HERTZ)
Backbore

(b)



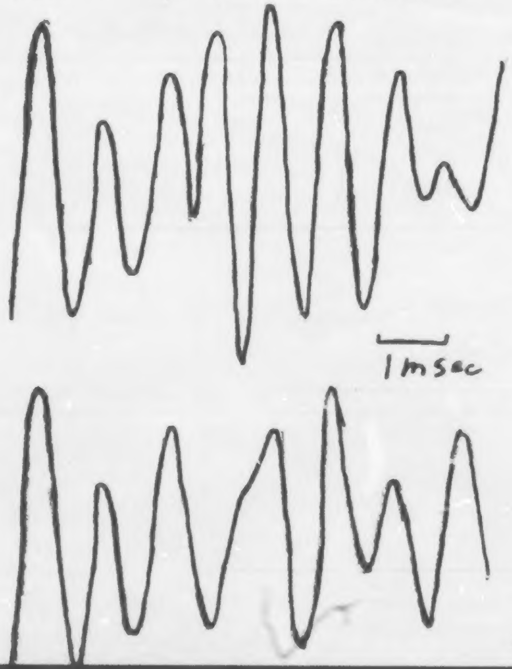
same, outside bell

appears like b. Note, if we regard (a) as a hill, (b) is the slope of this hill - first up (positive) then down (negative). This accounts for the characteristic oscillating wave form of both the trumpet and trombone. The oscillation must be attenuated by 6db per octave for exact differentuation to take place, and in the region of relatively low frequencies, this attenuation is reached, although high frequencies are radiated well.

If we now go back and examine the oscillations in the trumpet cup, we see that the vibrations of the lip provide many high frequencies. The trumpet acts as a resonator, accepting and amplifying those frequencies to which it responds, and rejecting others. This is why the timbre of the mouthpiece, sounded alone, is not so pleasant as the sound of the trumpet itself.

Now examine the oscillations in the small microphone which is inserted in the bell. Photographs taken 1 inch apart are shown. The changes arise mostly from oscillations whose wavelength is on the same order as this distance.

The trumpet is so arranged that, whereas the fundamental is not well radiated, and hence exists as a standing wave in the tube, the higher harmonics travel straight through and are radiated - therefore they exist more as traveling waves than as standing waves. Hence, if the oscilloscope is triggered by the fundamental, as the microphone is slowly withdrawn, the higher harmonics give the appearance of moving along the fundamental as shown, and decreasing and increasing in amplitude as we pass nodes and antinodes of this shorter wavelength.



The same note, recorded on the miniature microphone at points one inch apart in the last section of tubing.

Some part of these higher harmonics is radiated, which is why we have standing waves too. For example, the distance between a pressure node and antinode for the tenth harmonic of middle C will be about 1 1/4". Hence, this is the distance between the maximum and the minimum for oscillations of this frequency. This will be superimposed on the traveling wave, whose amplitude will oscillate with this period.

Quantitative:

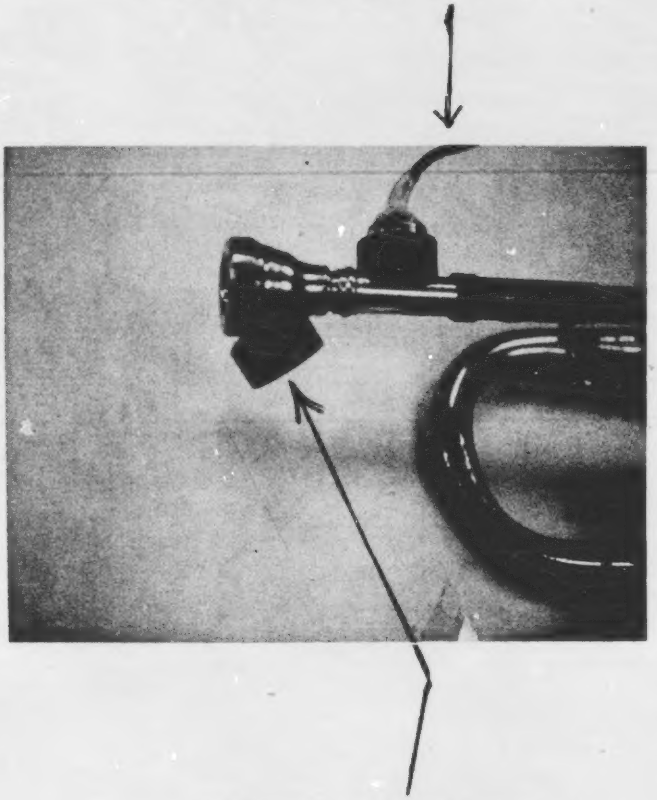
Quantitatively, we must analyze the pictures of the oscillations in terms of the harmonics. This requires enlarging the picture using an opaque projector, and drawing over the enlarged waveform, measuring the ordinates for a sequence of abscissa values, and using a calculator or computer to measure the harmonics. A second way is to record the tones on a tape loop, and use a frequency analyzer, such as the Pasco Model 9302 to obtain the harmonics digitally. Values obtained for the amplitude of various harmonics are shown in Figure 3. It should be possible to use such values to tell the difference between a poor and a talented trumpeter, and perhaps to tell one how to improve. The timbre or quantity of the harmonics present is also a function of the loudness of the tone. For example, a player blowing fortissimo raises larger quantities of the higher harmonics than blowing piano - soft tones contain primarily the fundamental. This is explained by Benade in the Scientific American (July, 1973 p. 24). If we look at the quantity of harmonics present, it is important to realize that the trumpet cannot add anything to the cup oscillation. It takes the oscillations already present in the cup, and, because of its own resonant nature, acts as a filter, letting only certain frequencies through. Any region it lets through is called a formant, because it imposes on the waveform already present, its own resonant pattern. For example, in the trumpet, the first few harmonics are more strongly resonant than the higher frequencies. This is the picture in the backbore. Outside, the bell acts as another filter, reducing the lower harmonics, with the result that the third, fourth, and fifth harmonics are strongest in a trumpet. A larger bell would allow lower frequencies to be emitted.

Results:

Obtain the ratios for the harmonics in the backbore and outside the bell. Plot the ratio as a function of frequency. This shows that the radiating efficiency of the bell changes with frequency.

FIGURE 1

microphone attached to backbore



fitting for microphone
on cup

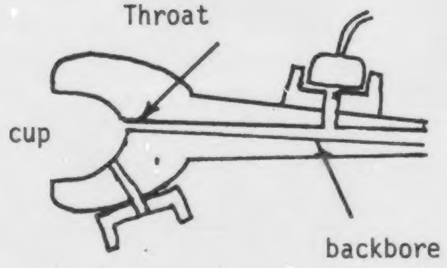
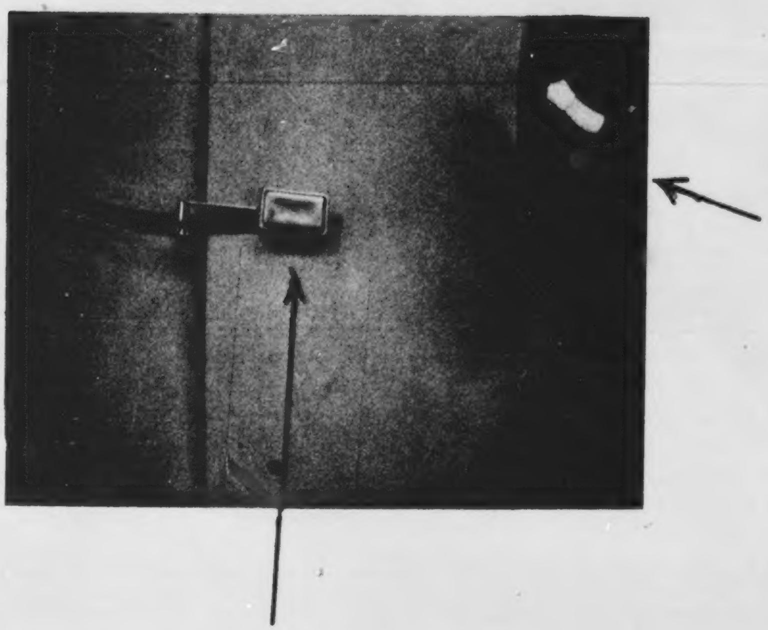

Throat
cup
backbore

FIGURE 2


trumpet bell

miniature microphone General Cement
1/2" x 5/16" x 3/4" catalog No. Q4-191

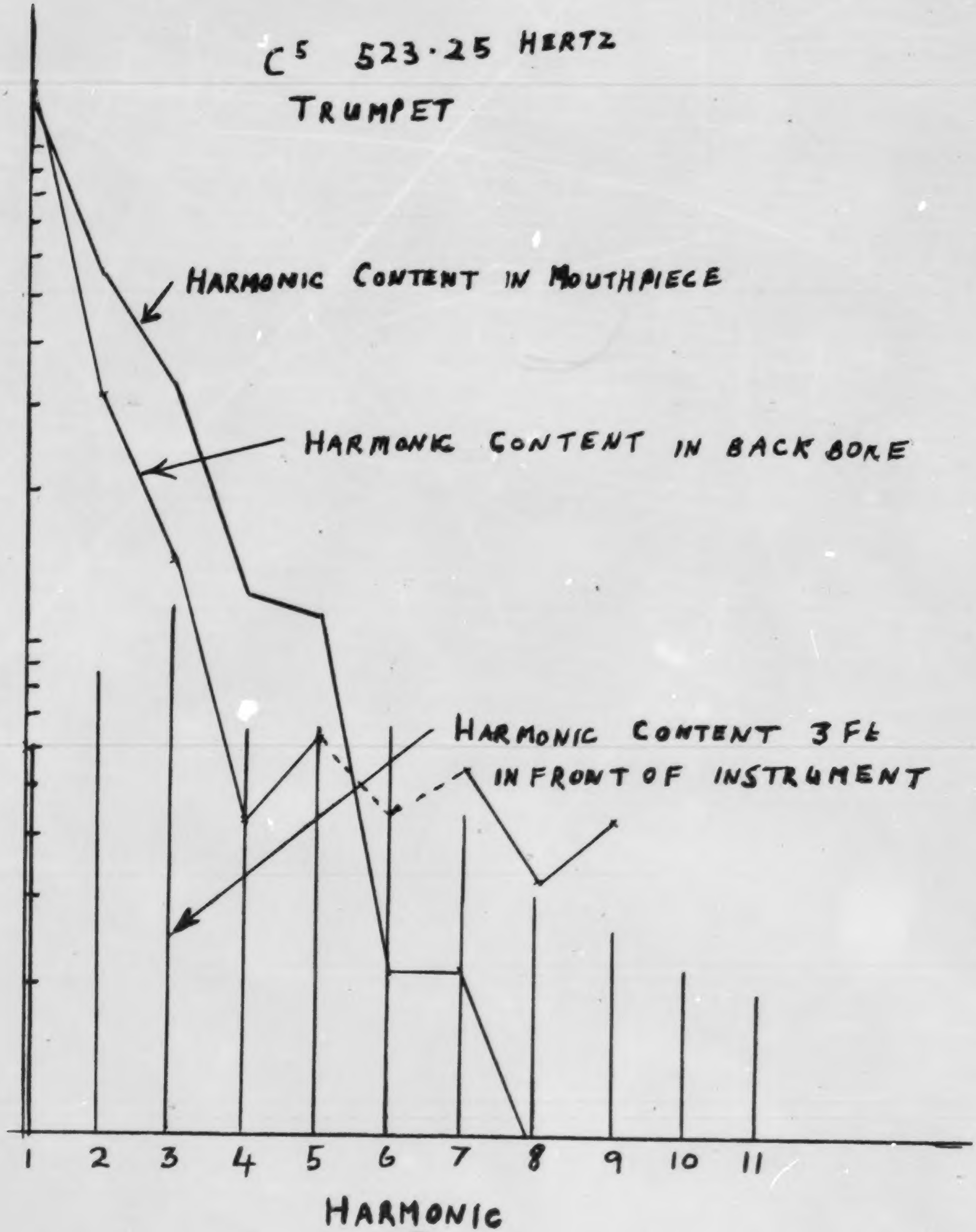
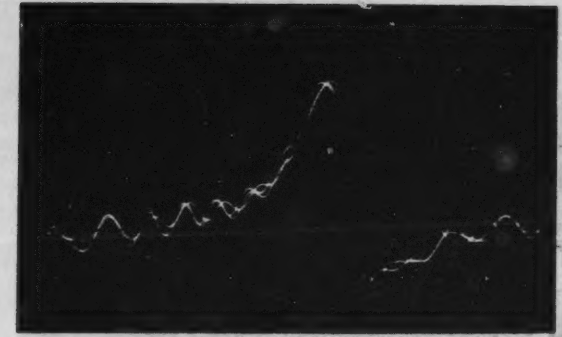
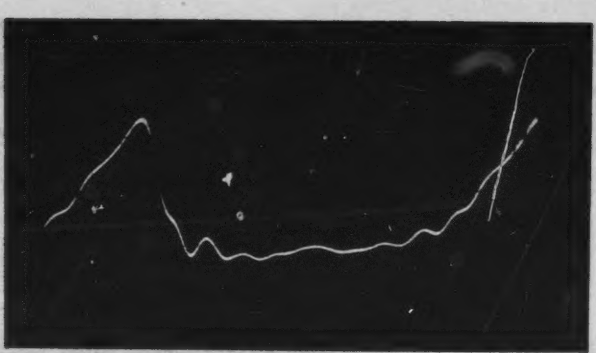

C⁵ 523.25 HERTZ
TRUMPET
HARMONIC CONTENT IN MOUTHPIECE
HARMONIC CONTENT IN BACK BORE
HARMONIC CONTENT 3 Ft
IN FRONT OF INSTRUMENT
1
2
3
4
5
6
7
8
9
10
11
HARMONIC

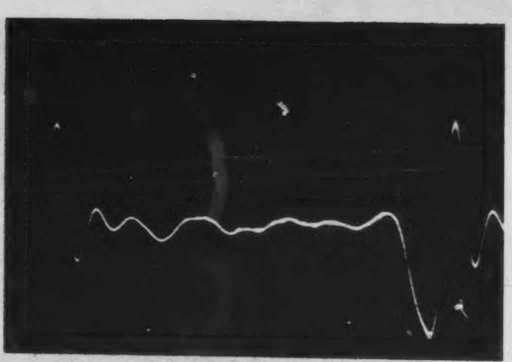
Figure 3



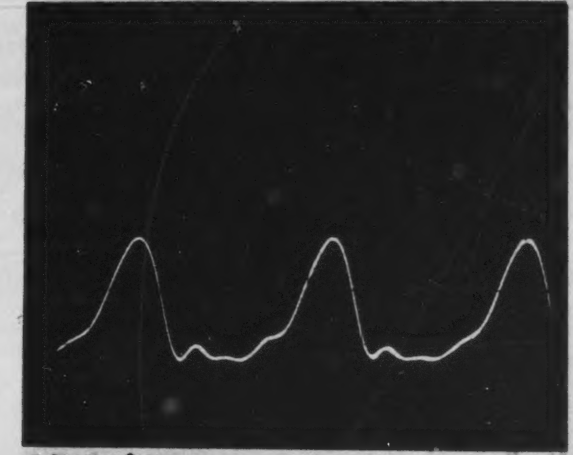
MIDDLE C TRUMPET MOUTHPIECE



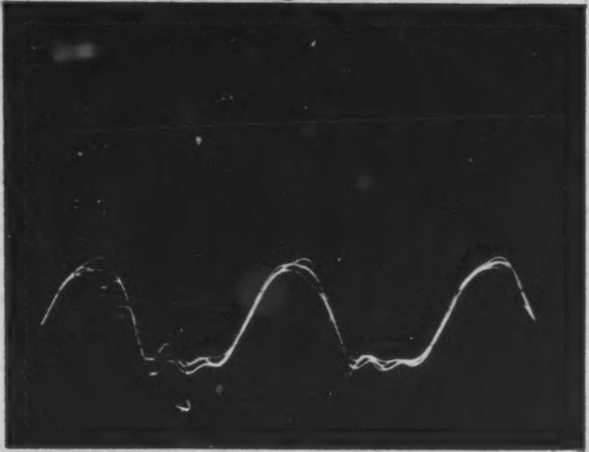
C BACK BORE



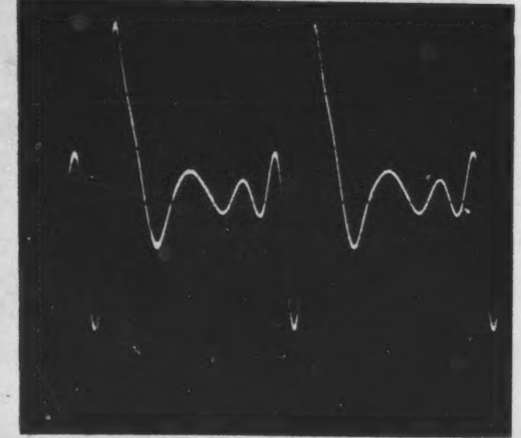
C BELL



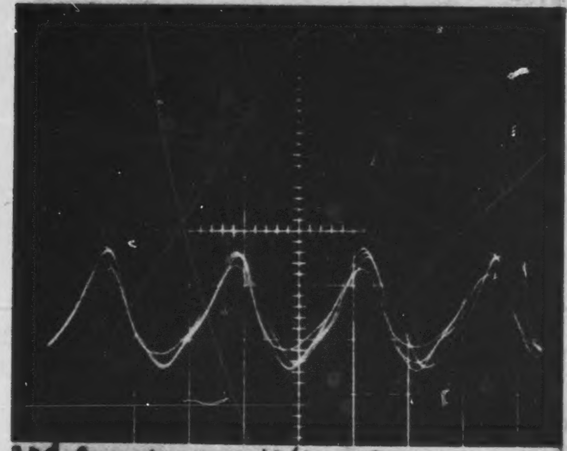
MOUTHPIECE C ABOVE MIDDLE C



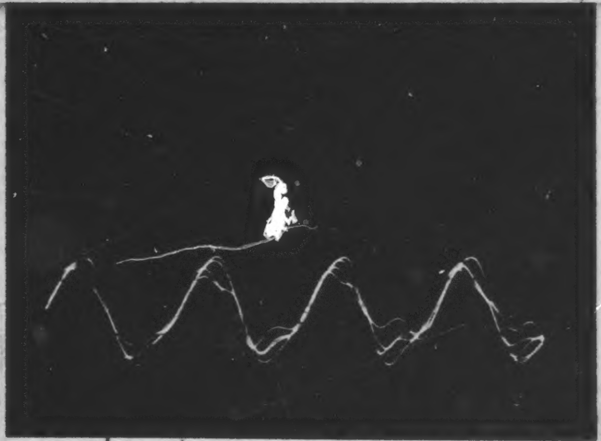
C ABOVE MIDDLE C BACKBORE



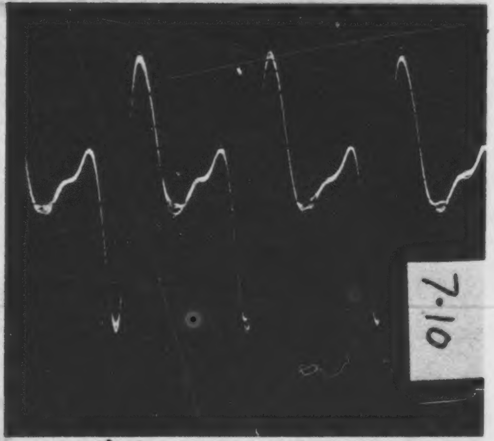
C ABOVE MIDDLE – BELL



2nd G above middle C



2nd G above MIDDLE C BACKBORE



2nd G ABOVE MIDDLE C BEL

OSCILLATIONS IN CYLINDRICAL AND CONICAL TUBES

Object:

To compare the timbre and pitch of notes obtainable by exciting air vibrations in conical tubes, and cylindrical tubes closed at one end.

1. Oscillations in a cylindrical tube closed at one end.

Many musical instruments (trumpet, trombone, clarinet) are pipes of nearly uniform bore, but modified by a taper, bell, and mouthpiece. To avoid such complications, a pipe of thin wall conduit, such as electricians use (see Benade "Horns, Strings and Harmony"), half an inch or a little more in diameter and about four feet long is suitable. A length of 50 inches should give middle C for its third harmonic, which is convenient. Alumimum pipe is lighter to handle. A collar, as shown in figure 1 surrounds the end of the pipe to act as mouthpiece or the pipe end is merely dipped in epoxy resin. The frequency of possible modes excited by blowing this device in the same manner as a trumpet, should be a series of odd multiples of the fundamental, the odd harmonics.

This contrasts with a pipe open at both ends, such as an open organ pipe, flute or the "corrugahorn" described by Crawford in the American Journal of Physics for April 1974 (vol. 42, p. 278), which give all multiples of harmonics, as does a conical instrument such as the bugle or oboe.

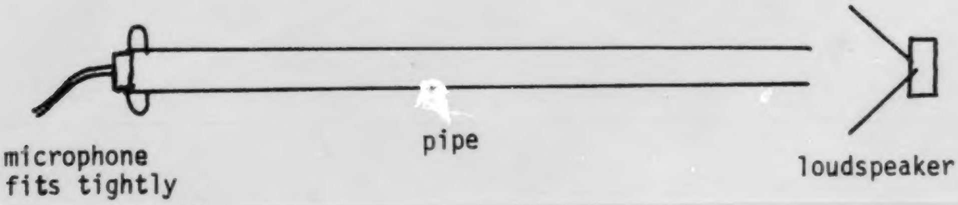
The pipe is blown like a trumpet, and the frequency obtained by comparison with an oscillator - speaker combination tuned to the same pitch. At least three modes should be capable of excitation. The differences between these values and those calculated are found on the assumption that the fundamental mode corresponds to a quarter wave length for the length of the pipe. End effect corrections (obtained by adding 0.3 diameter of the pipe) can be applied.

Hence,

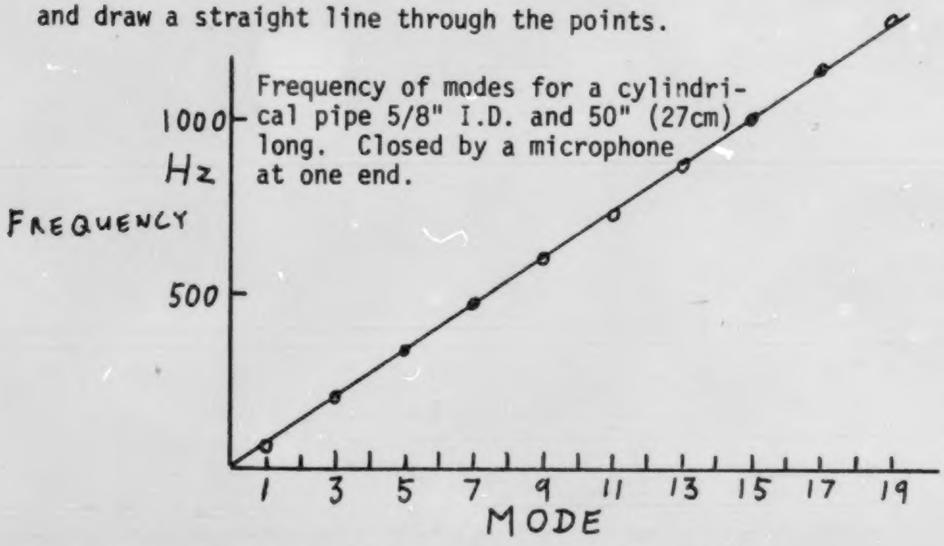
$$\text{frequency} = \frac{1100}{4 \times \text{length of pipe in feet}}$$

The timbre of a tubular pipe differs from that of a cone by again containing only odd harmonics. It is interesting to see how different mouthpieces - trombone, trumpet, cornet, affect the timbre. The mouthpiece fits into a brass tube turned to fit the end of the pipe. A microphone connected to an oscilloscope conveniently shows this, and photographs of the trace shows that the trombone mouthpiece gives fewer high harmonics than a cornet mouthpiece.

The resonant frequencies of the tube can be examined by attaching a microphone tightly over one end - there must be no air leaks past it - and exciting the tube by placing a loudspeaker connected to an oscillator at the other end. On hitting a resonant frequency, a large output from the microphone is seen, if the microphone is connected to an oscilloscope, or heard if connected to an amplifier and speaker.



Plot the resonant frequency against the odd whole numbers, as shown below, and draw a straight line through the points.


Frequency of modes for a cylindrical pipe 5/8" I.D. and 50" (27cm) long. Closed by a microphone at one end.

MODES OF OSCILLATION OF A CONICAL PIPE

Object:

Common instruments having a conical bore are the bassoon, oboe, and bugle.

It is possible to examine the natural modes of a conical pipe employing a cavalry bugle, such as is used for army bugle calls. The tube of the instrument itself is only about 45 inches long, but it forms part of a cone which would be about sixty-six inches from the point or apex of the cone to the bell of the bugle which it forms. Such a cone has a lowest vibrational mode near 100 Hertz, since the cone must be twice as long as a cylindrical pipe closed at one end having the same fundamental frequency. The cone and the cylinder are unique in having modes which are simple multiples of the fundamental.

It is preferable to employ an exact cone, for which calculations can be made. The Shakespeare Company of Columbia makes fishing rods and broadcast antennas by winding fiberglass on a conical metal mandrel. The inside of the fiberglass cone is perfectly smooth. Any such cone is suitable, provided the wide end is more than three times the diameter of the narrow end. The air in the missing third is so "stiff", its presence or absence does not alter the harmonic structure appreciably. We employed a cone 100 inches long, tapering from 1/2 inch in diameter at the narrow end to 1 9/16 inch at the wide end. If the length of pipe is L inches, the narrow end has a diameter a, and the broad end b, the "pedal tone", the lowest natural note, is given by

$$12 \times \frac{1100}{2 \times L} \ (1 - a/b) \ = \ \frac{6600}{L} \ (1 - a/b)$$

Here $\frac{L}{1 - a/b}$ is the length of the cone from apex to bell, in our case 147 inches, giving approximately 45 vibrations per second. It is difficult for the mouth to reach such low frequencies simply by blowing into a half inch tube. The tube has a collar as shown (figure 1) surrounding the narrow end, to aid in blowing the tube with no mouthpiece. Alternatively, the small aluminum tube turned to fit the outside backbore of a trumpet or trombone mouthpiece seen in the figure can be inserted. The higher notes of this system can easily be reached either with the open tube, or a trumpet mouthpiece. The lower notes require a trombone or tuba mouthpiece. It is useful to have available cornet, trumpet, and trombone mouthpieces. The resonant frequency of the system is not appreciably altered by the mouthpiece, but there is a considerable change in timbre. The frequency can be "lipped" lower.

Results:

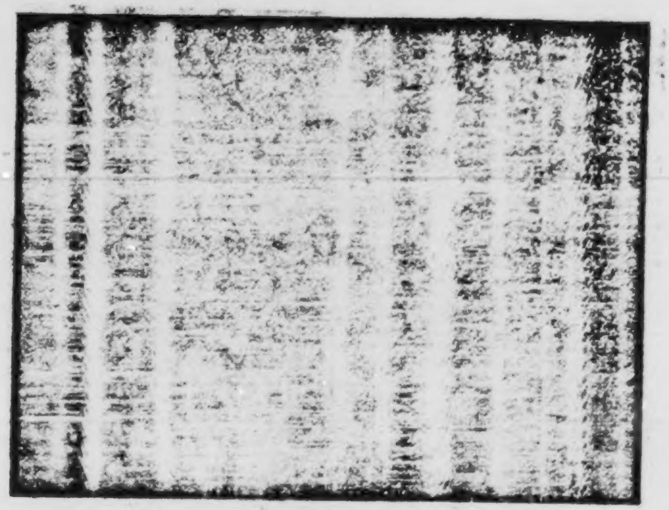
The resonant frequencies of the system are found experimentally by blowing, and comparing with an oscillator and loudspeaker. Alternatively, a microphone may be attached to an oscilloscope, and the oscilloscope photographed using a known time base to determine its frequency. The time base should be calibrated by feeding the microphone with a tuning fork. The experimentally determined frequencies are compared with the harmonic series. All should be present.

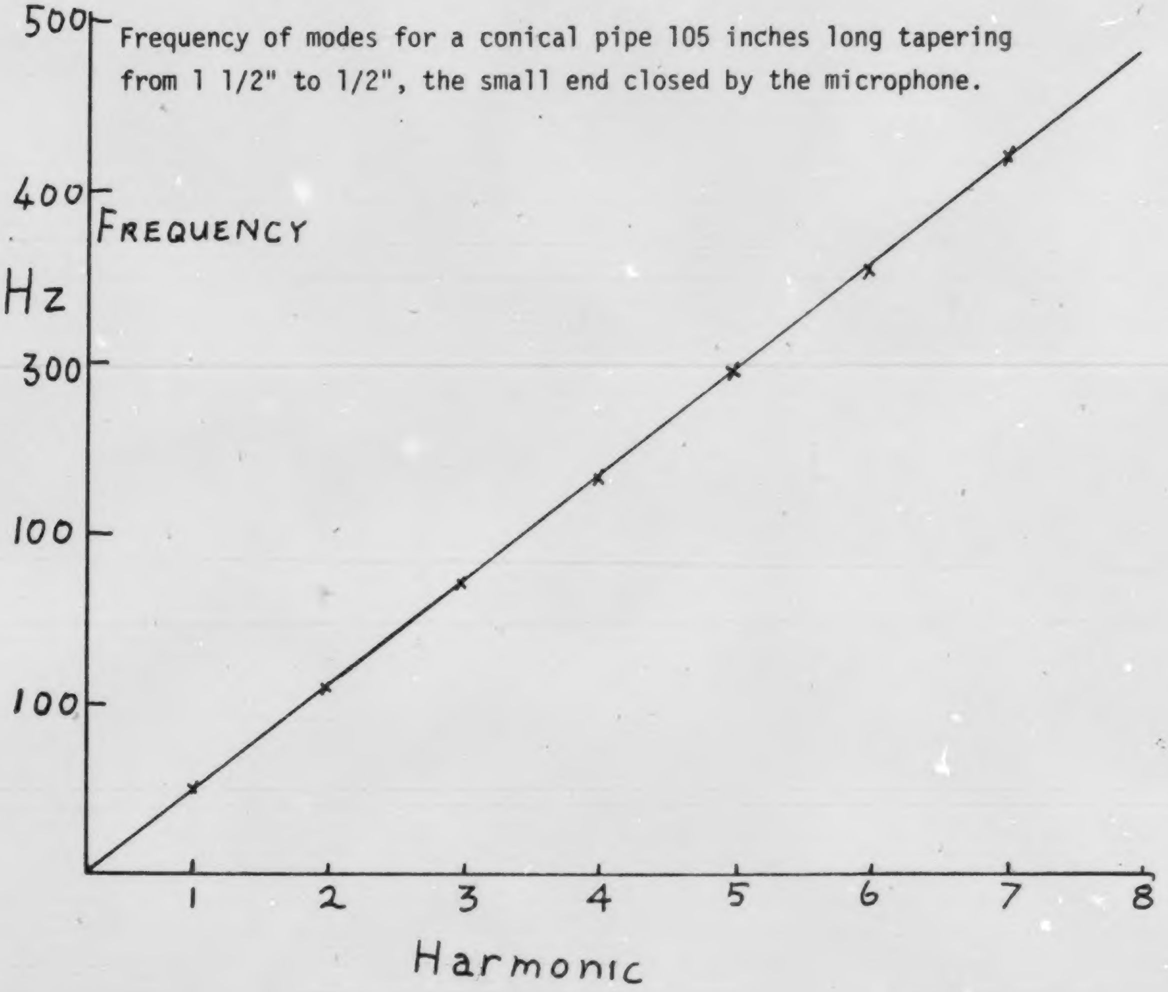
One may reverse this process, and very simply, apply an ear to the small end of the cone, much as an ear trumpet, and hear a maximum response at certain frequencies of the oscillator and loudspeaker, placed just outside the large end of the cone. For our cone, the experimental resonant frequencies were found to be 49,109,168,230,225,270,359,406. As shown in the figure.

The experiment may be improved by examining the oscillations by a microphone sealed tightly over the small end of the pipe, excited by a loudspeaker placed over the large end. If the frequency emitted by the speaker is varied, and the amplitude kept constant, the cone will resonate to certain well defined frequencies which can be seen by applying the microphone output to an oscilloscope.

All harmonics can be excited simultaneously by blowing the cone like a horn, as an analysis of the oscilloscope trace shows.

FIGURE 1





Frequency of modes for a conical pipe 105 inches long tapering from 1 1/2" to 1/2", the small end closed by the microphone.

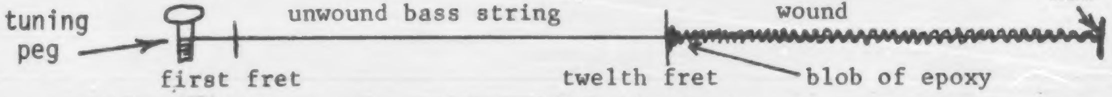
WAVES ON STRINGS - THE GUITAR

Object - To find the dependence of frequency on

1) The length of the string, and to find the velocity in that string
2) The tension in the string
3) The mass-per-unit-length of the string

Some other observations on guitar resonances will also be made.

Apparatus - The modified guitar, a meter ruler, a calibrated oscillator and loud speaker. The guitar has the base string unwound from the tuning peg to the twelfth fret. A blob of epoxy resin should be placed to prevent the wound wire unwinding further, before cutting at the twelfth fret.



Method - 1) Measure the distance between the top 12 successive frets and the lower nut,over which the string passes for the highest E string. Find the ratio of the distance between successive frets and the distance of the upper of the two frets to the lower nut. This should be a constant, approximately 1:18 (the so-called rule of 18). More accurately it is 1:17.817. Now find the frequency for fingering successive frets by comparing the plucked string with the oscillator. Divide the frequency by the length of string which is vibrating. Is this constant for all the frets you tried? What does this show about the relationship between the length of the string and the frequency? The length of the string is one half wavelength for the fundamental mode. Pluck the string, then lightly touch it at the halfway point (the 12th fret) and note the pitch jumps an octave. Why? Find the velocity of waves on the string from the formula

Velocity = Frequency x wavelength

Note that the twelve frets give all the full and half notes on the tempered scale for the octave, and that the ratio and not the difference in frequency is constant. If the scale depended on a constant frequency difference, the separation between frets on a guitar would always be the same.

2) Hooke's law states that the tension in a wire is proportional to the extension of the wire (see fig.1). Hence, the tension in the guitar string is proportional to the number of turns of the tuning peg. Rotate the tuning peg in quarter turns, by three or four turns, on one string, and find the frequency for each position by using the oscillator. Then plot the square of the frequency against the number of turns of the peg. Is this a straight line? What does this mean?

3) The bass string on the guitar is normally wound with wire, to increase its mass per unit length without increasing its stiffness. On this guitar, the string has been unwound to the halfway point. Pluck it, and note the peculiar timbre. This is because the normal modes of a uniform string are harmonics, i.e. multiples of the fundamental frequency. No such relation exists for a non-uniform string, so it does not sound so pleasant, although perhaps it could be used for its bizarre effect.*

Press the string to the fret at the halfway point, and find the frequency for the top half of the string, and for the bottom half. Since the two halves have the same length and tension, the difference in frequency arises from the difference in mass per unit length only. The mass per unit length for the wound string is $7{\cdot}51 \times 10^{-2}$ gm/cm, and for the unwound $1{\cdot}5 \times 10^{-2}$ gm/cm.

We wish to relate mass per unit length m to frequency f As the mass per unit length goes down, the frequency goes up-which is why the high guitar or violin strings are the lightest, so we might have

$$\text{frequency} \quad f \propto \frac{1}{m} \text{ or } f \propto m^{-1}$$

Other possibilities are

$$f \propto \frac{1}{\sqrt{m}} = m^{-1/2}$$

---

* A most interesting article on the calculation of the normal modes of such a string and their experimental verification, is given by T. D. Rossing, Am. Jour. Phys. 43, 735 (1975).

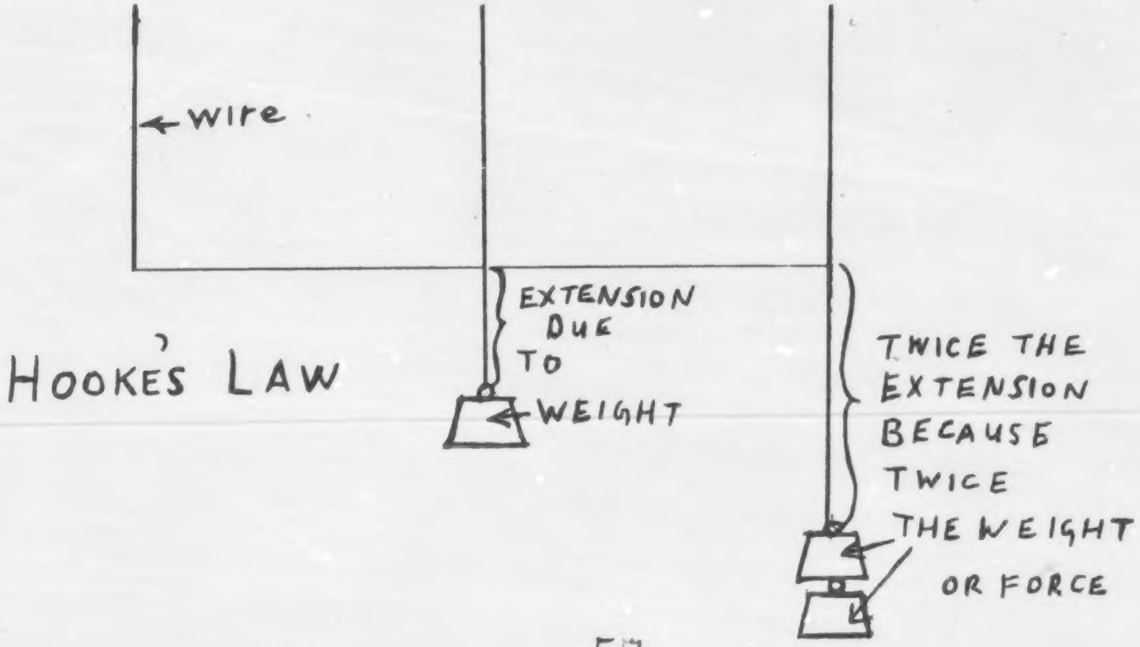
Both these are given by $f \propto m^n$ where we wish to find n

$$n = \frac{\ln f_1 - \ln f_2}{\ln m_1 - \ln m_2}$$

where f_1, f_2 and $m_1 m_2$ are the frequencies and mass per unit length of the two strings. You should, by now, have already found that for a fixed length of string the frequency, and hence, the velocity, is proportional to the square root of the tension. Use the results of the mass per unit length to confirm the formula for the velocity v of waves on strings

$$V = \sqrt{\frac{\text{Tension}}{\text{mass per unit length}}}$$

The guitar has several resonances associated with the instrument. Blow gently across the air hole, and try to find the frequency of the air in the body of the instrument. Then tap the top of the guitar. Do these two resonances lie within the range of the instrument?
(This is E_2, 82.4 Hertz to about E_5, 659.2 Hertz).
Notes near these resonances tend to be unduly amplified.


wire
EXTENSION DUE TO WEIGHT
HOOKE'S LAW
TWICE THE EXTENSION BECAUSE TWICE THE WEIGHT OR FORCE

THE VELOCITY OF SOUND

Several methods exist to measure the velocity of sound using traveling waves. Three simple methods of measuring the velocity by using the wavelength are given here. All require an oscillator connected to a loud speaker. A 600 ohm to 8 ohm transformer is normally required between oscillator and speaker. The oscillator is calibrated by beating against a suitable tuning fork, say C_6, 1046 Hz. (concert pitch).

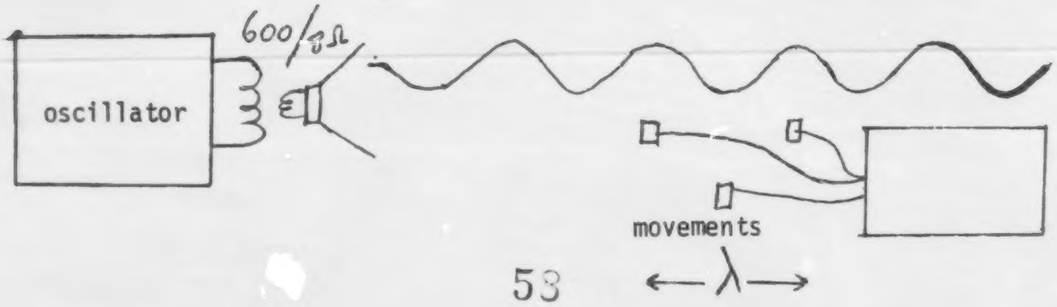
A) Requires only a stereo amplifier, two microphones and a pair of head phones.

B) Requires one microphone and a single trace oscilloscope.

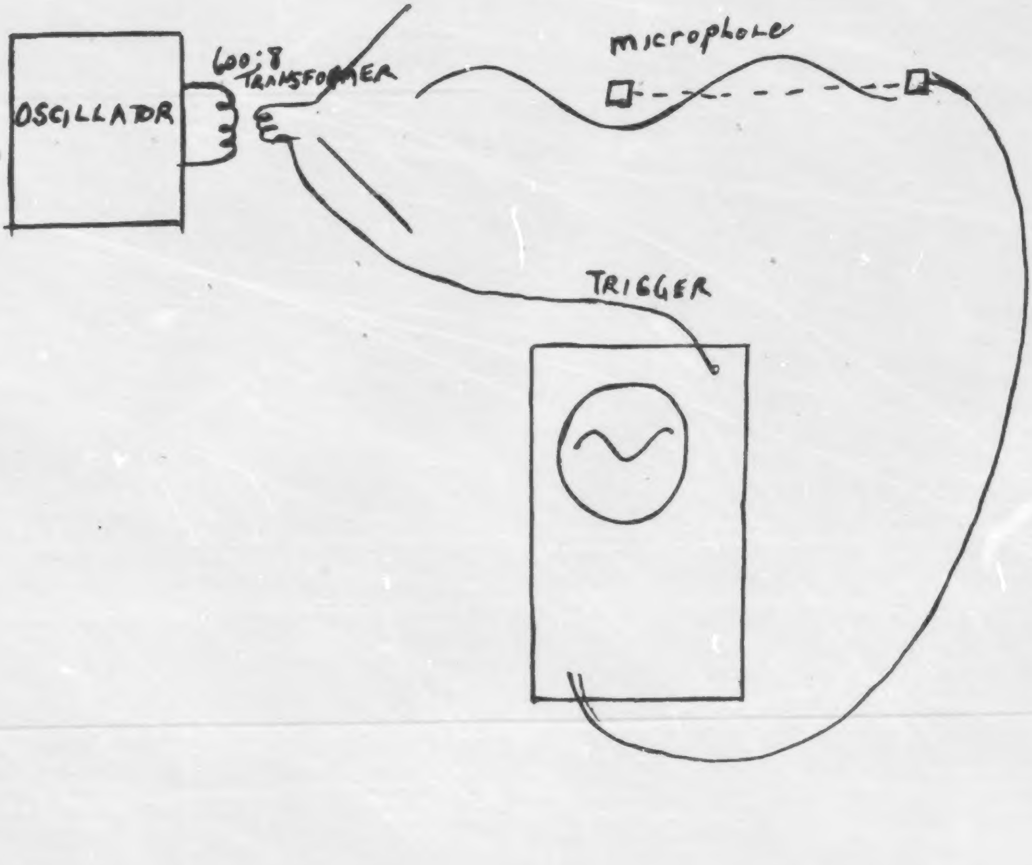
C) Requires two microphones and a double trace oscilloscope.

A) Procedure.

Connect the two microphones to the stereo amplifier, plug in headphones and turn off the amplifier speakers output. Turn on the oscillator, and set it at a suitable value, say 5kHz. Put the amplifier to mono, place the microphones together about two feet from the speaker, hooked to the oscillator. Then move one forward to the position quietest in the head phones and back to a similar position behind the other microphone. The distance moved by the microphone is the wavelength. The velocity of sound can be found by multiplying the wavelength by the frequency.



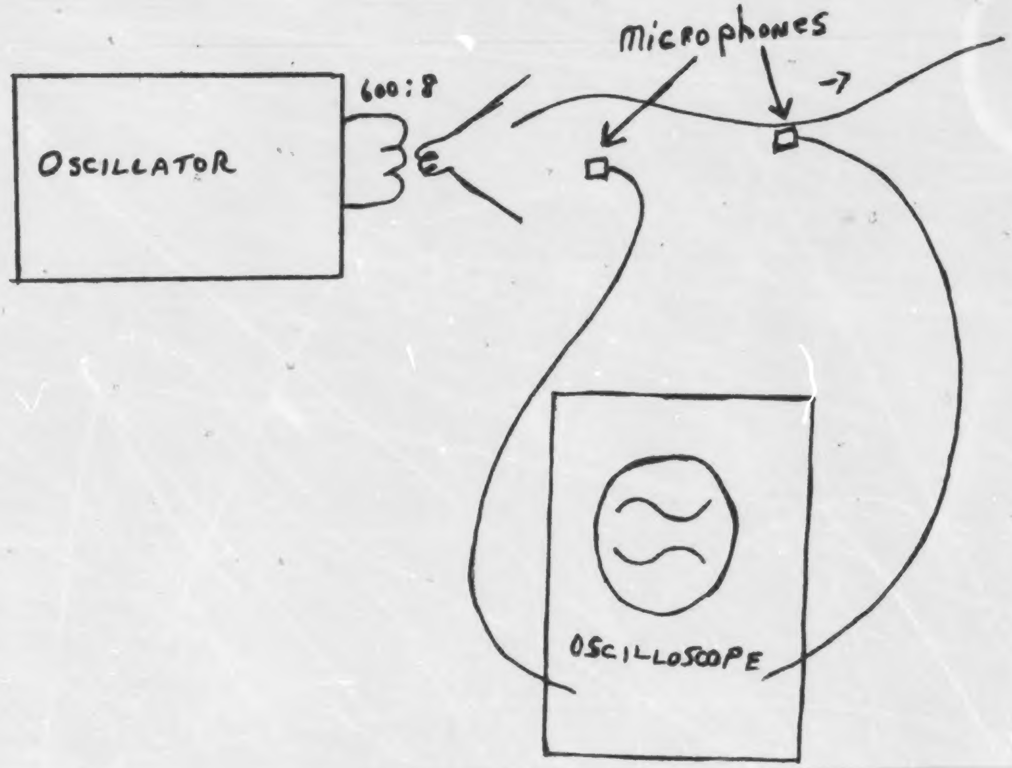
B) Connect the microphone output to the oscilloscope input, either directly, if it is a high impedence mike, or through an amplifier or transformer if of low impedence. Connect the speaker output from the oscillator to the "external trigger" of the oscilloscope as well as to the speaker. Set the oscillator to a suitable value, say 1 kHz, and the oscilloscope time base to, say, 1 millisecond per centimeter. Note the distance the mike must move, so that an oscillation on the oscilloscope screen drops back by exactly one period. This corresponds to one wavelength in air.



C) Connect the two microphones to the two oscilloscope inputs, either directly or via a stereo amplifier. Internal trigger and a time base of, say, 1 m sec/cm and rquired. Set the oscillator amplifier at, say, 1 kz, the two traces on the oscilloscope move as one mike is moved relative to the other. A motion of one wavelength of one microphone causes the corresponding trace on the oscilloscope to drop back by one period. Hence the wavelength may be determined, and, since

Velocity = frequency X wavelength

to velocity of sound in air may be found.



NOTE: it is possible to use a single pulse for this experiment, by switching a 6 volt battery across the speaker in place of the oscillator. No wavelength is then required, but an accurately calibrated oscilloscope time base.

Room Acoustics - Reverberation

Objective: Investigate the acoustical properties of selected rooms. Include the properties of rooms suitable for speech, voice and orchesteral works. The acoustics of a church would be a suitable subject for investigation. The acoustics of a room depend on the properties of the shape of surfaces and the absorption of the surface. Determine reflections and reverberation times for the rooms.

1. In this part of the experiment use a model to see how sound spreads and is reflected through a room. Use a ripple tank and place a metal strip bent to the roof line and another (in a separate experiment) bent in the plan view of the room. A wave is started from the position of the performance and observed as the wave spreads to the audience. This scale model method can help in determining the presence of echos. The waves can be initiated simply with a finger or with more precision with the vibrator supplied. Examine the effects of reflection and diffraction.

   If the room design has a strong echo see if reflectors can be added to reduce the effect of the echo.

2. Reverberation: When the echos from the various surfaces in a room occur so closely spaced in time that the individual echos cannot be detected the room is reverberant. The time for the sound level to decay or drop to inaudibility is the reverberation time. For the sake of reproducibility and to define inaudibility more exactly the reverberation time is the time required for the sound intensity to fall to one millionth of its initial value.

   To measure the reverberation time of a room, the simplest method is to fire a shot from a starter's pistol or burst a balloon at the location of the performance. The time for the sound to decay to inaudibility can be timed with a stopwatch.

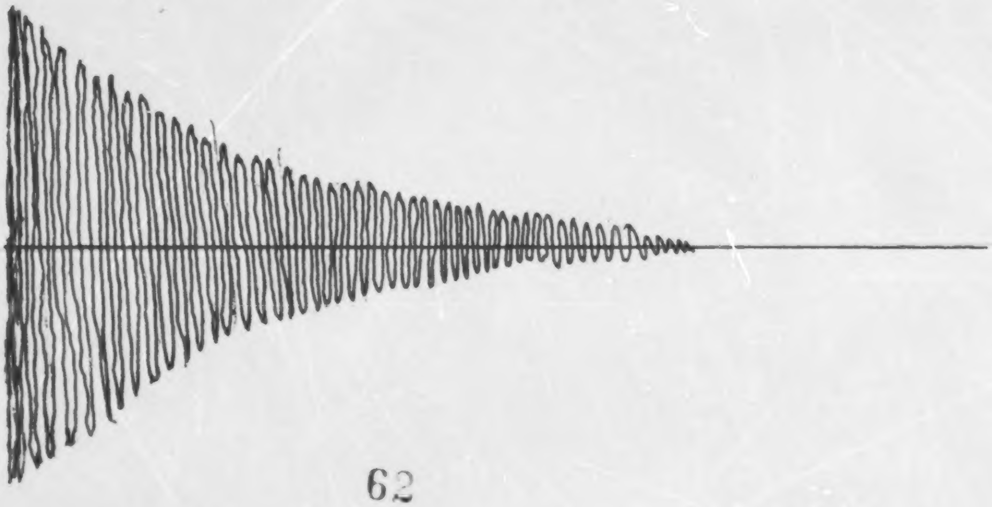
   A more exact method is to record the sound on a tape at 7 1/2 inches per second. This can be analyzed in the laboratory by listening to the tape or by electronic means. The student will conduct a series of measurements by several methods and compare the results. The first method is the least accurate. In the next parts of the experiment the tape speed is reduced to improve the accuracy of the measurement. The subjective reduction in pitch does not influence the measurement. The sound burst contains a great number of frequencies. Any frequency dependent effects in the threshold of hearing curve do not influence the result since the final inaudibility criterion is still valid. Because of interference from room noise it is important to have a "quiet room" for this experiment.

a. First the tapes can be run at 7 1/2 inches per second and the time for decay measured in five trials. Give the average value of the reverberation time obtained for the rooms in which the tapes were prepared.

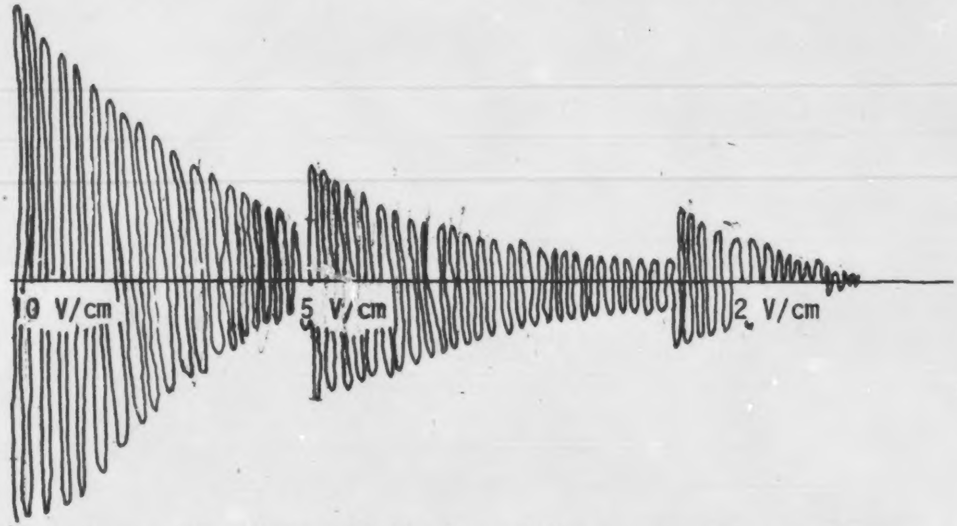
b. Run the tape at 1 7/8 inches per second and repeat the estimation of the time with a stopwatch for 5 trials. Report the average values.

c. Use an oscilloscope to estimate the time for the sound to decay. The sound intensity in a room with reverberation decays exponentially, so if a plot is made of the logarithm of the amplitude versus the time is made for the sound intensity decrease a straight line should be obtained. The time for the amplitude of the signal on the oscilloscope to decrease by 1/1000 gives the reverberation time. The reverberation time is the time for the intensity to decay by a factor of 1000000, i.e. the intensity changes by -60 dB. It is also the time for the amplitude to decay by a factor of 1000 because the intensity is proportional to the square of the amplitude. The oscilloscope is used in a way that measures the amplitude of the electrical signal from the microphone. The microphone used was sensitive to pressure variations. The square of the pressure variations. The square of the pressure variations is proportional to the intensity of the sound. Also, the voltage produced by the microphone is proportional to the intensity of the sound.

Attach the tape recorder to the oscilloscope through the amplifier. The time base should be set at 1 or 2 seconds per cm. Observe the oscilloscope as the balloon is burst. Run the tape recorder at 1 7/8 inches per second. A trace of the following sort should be seen.



Measure the height of the pulse h, as a function of position on the trace (time), plot this on logarithmic paper (3 cycle) and hence find the time it takes to decay through three decades. To facilitate these measurements, the gain of the oscilloscope must be changed as the sound is decaying thus.



This way the pulses will not be off the top of the screen for very long at the beginning, or too small to measure at the end. Do not be concerned if the initial burst of sound is not on the screen of the oscilloscope. When you have mastered this, photograph the picture using the polaroid camera, setting the exposure on "time" or "bulb" at f4, and measure the photograph with a transparent ruler. Then plot the results on the logarithmic graph paper. The graph should require 3 cycles vertically and 10 divisions for 10 seconds horizontally. The voltage change you measure may not cover all three decays of the graph paper. The points should give a nearly straight line. By eye and with the aid of a transparent ruler draw the best curve through the data points. The extension of this line to the time axis gives the reverberation time. The decay wave should be very nearly exponential. Any deviation can be the result of echos or flutter echos.

In place of the oscilloscope and camera, it is possible to use a strip chart recorder. The output from the amplifier is fed to the recorder via a diode (we used Motorola Hep-170 but almost any diode will do). Provided the tape recorder is run at its slowest speed and the chart run at a reasonably fast speed, a few inches per second, the decay of the sound will be plotted and can be measured.

It is possible to calculate the reverberation time from parameters which are known or can be estimated. The dimensions of the rooms measured have been given on the data sheet. Compute the reverberation time for each room.

Data Sheet - Sample

	Area	Absorption	Sa
Floor			
Back Wall			
Ceiling			
Front Wall			
Side Walls			
Occupants			
Furnishing			

Sum of Absorption Factor (Sa) =

Volume of Room =

Reverberation Time = $0.009 \frac{\text{Volume}}{\text{Sa}}$

Compare all the reverberation times. Which method is more accurate? Why? Account for any difference between the calculated reverberation time and the measured time.

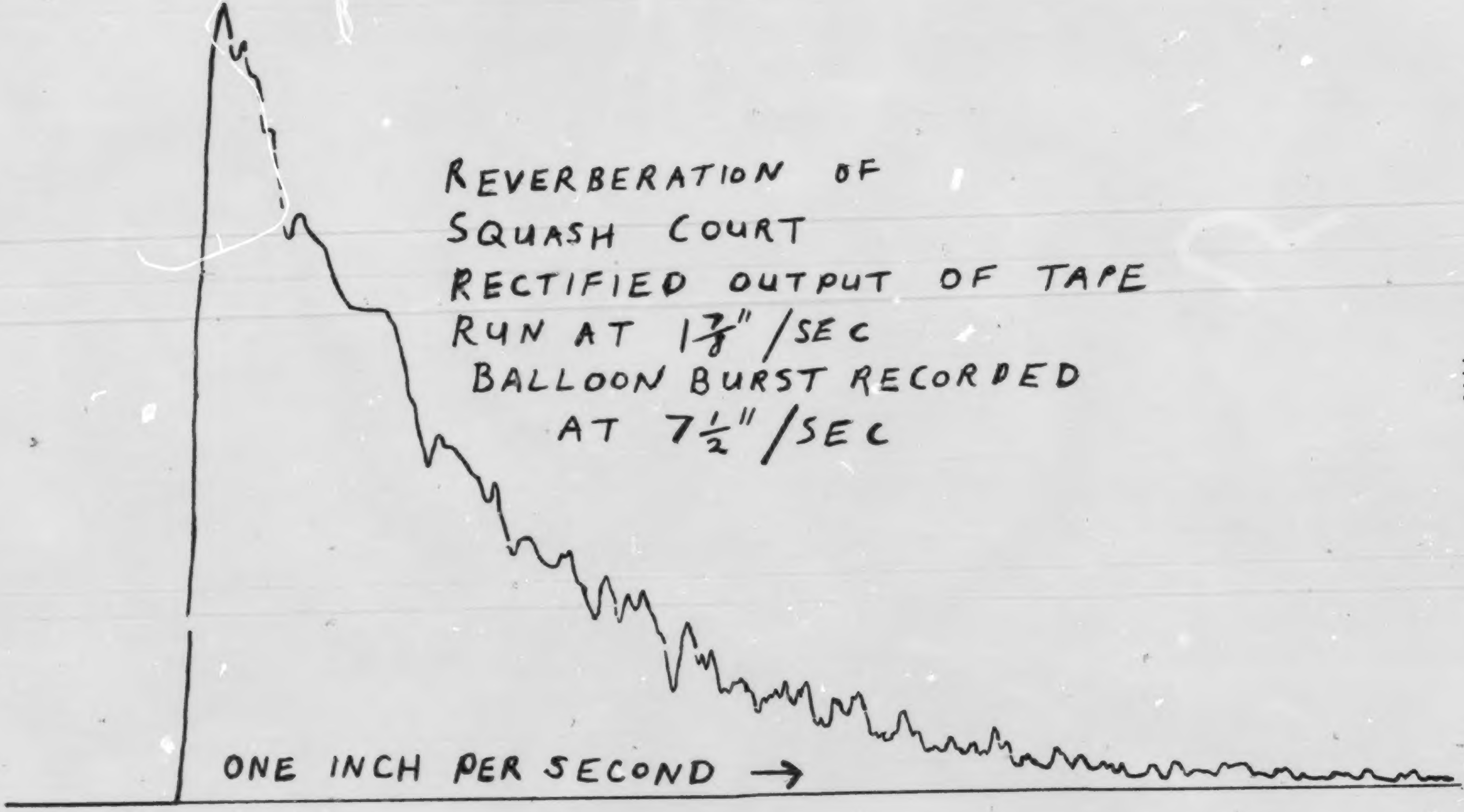
3. Sound Intensity in a Reverberant Room: One further quantity is of acoustic interest. If an instrument or sound is made of the same intensity in several rooms the subjective and measured loudness will be higher in the reverberant room. Make a recording of a pure tone feeding the same power to a loud speaker placed successively in several rooms. Measure the relative amplitudes on the oscilloscope. Also measure the intensity using a sound level meter on the C scale or flat response mode. The relative response levels can be compared to the smallest level. These response levels do not depend on the volume of the hall, but inversely on the area of absorptive material in the expression-

$$\text{Intensity level in hall} \propto \frac{\text{power output of instrument}}{\text{Sa}}$$

where Sa is the absorptive area.

Check this result, by seeing if the ratio of intensities measured by the sound level meter, or using the oscilloscope (ratio of amplitude^2) is proportional to the inverse of Sa for the rooms. These measurements should all be made with a microphone sensitive to sound pressure changes.

A simpler method of demonstrating the drop in intensity level, is to observe the decrease in microphone output for a constant sound source when two or three, four feet by eight feet sheets of fiberglass acoustic material are introduced into a small room.

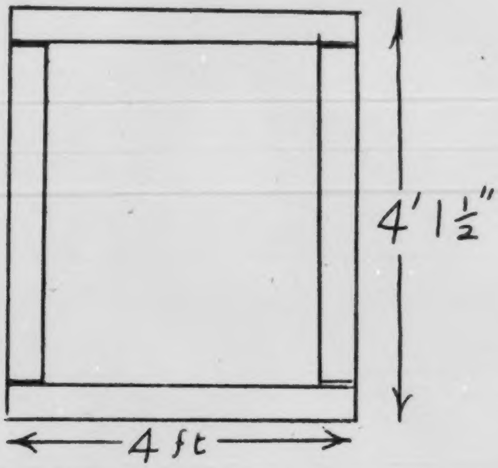


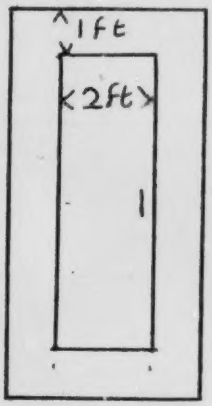
REVERBERATION OF
SQUASH COURT
RECTIFIED OUTPUT OF TAPE
RUN AT 1⅞"/SEC
BALLOON BURST RECORDED
AT 7½"/SEC

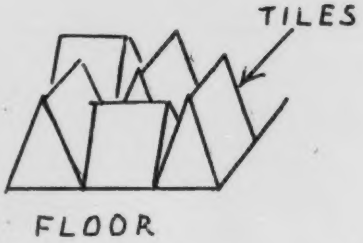
Fig 1

A Simple Anechoic Chamber -

The construction of the chamber from five 3/4" plywood 4'x8' sheets is relatively simple. Since two walls butt onto the other two walls as shown the floor must be 4' x 4' 1 1/2". This leaves the roof, of the remaining half sheet of plywood, 3" short, and a piece 4'x3" must be screwed on separately. The door should be cut from the center of the front wall as shown. When screwed together, the box is extremely rigid. The walls and floor were coated with contact cement, and fissured, painted, (Owens Corning) Fiberglass ceiling board, in 1 ft. square sheets, attached. These have a noise reduction coefficient of 0.8 - 0.9. A false floor of expanded steel was welded to a 3'10" square frame with 1 ft. high legs. Under this, more acoustic tile, glued in V shaped forms, was attached to the floor, and still more covered the ceiling. The V shaped construction absorbs low frequencies better, gives a larger absorptive area, and breaks up sound reflections.. More squares were attached about two inches off the walls, where space permitted. The whole idea was to incorporate as large a surface area of absorptive material as space would permit.







The principal interior equipment consisted of two 9" polyplanar speakers mounted approximately a little forward of, and level with the ears for a subject sitting on a chair inside the chamber. Cables from the speakers led to an amplifier external to the chamber, but a stereo T pad inside allowed the volume to be varied. In addition, a very valuable addition was a switch allowing the phase of one speaker to be reversed with respect to the other. A stereo headphone outlet was also attached, which can be used either for other speakers or a stereo headphone set. A cheap two-way intercom allowed communication with subjects in the chamber. A 60 watt incandescent lamp hung from the roof provided sufficient illumination. A microphone socket was also attached to the chamber wall, in order that instruments played inside could be recorded.

To insulate the chamber from noises outside, all cracks were filled with putty, glue or silicone cement. It is essential that this be done thoroughly. The complete outside of the chamber was covered with half inch thick felting, available from surplus, but 2 ft. by 4 ft. sheets of fiberglass would serve as well. The felting overlapped at the door, which was tightly sealed when shut. The box sat on rubber feet, about four inches thick.

The cost of the chamber was approximately	$150.00
The principal cost was 5 sheets of 3/4" plywood 6' x 8' at $13.76 each	68.80
220 sq. ft. of fiberglass tile at $.185 each	40.70
giving a total of	$109.50

Reduction in intensity of sound from outside transmitted into the box

1 kc	60 db
600 Hz	35.5 db
200 Hz	35.5 db

Ratio of sound intensity from one ear to the other ear with one loudspeaker on

10 kc	14 db
1 kc	12.3 db
500 kc	8 db

TRANSMISSION AND REFLECTION OF SOUND

Objective: Measure the transmission of sound by various materials.

Materials: Two cardboard tubes, loud speaker, signal generator, microphone, oscilloscope.

Method: Study of the reflection and transmission of sound by various materials. Two hollow cardboard tubes are used. One tube has a speaker connected to an oscillator and the other contains a microphone connected to an amplifier and then to an oscilloscope. Each tube is wrapped with sound insulating material. The oscilloscope is used to determine the amplitude of the received sound waves when various materials are placed for reflection before the open ends of the tubes or or when the material is placed in a gap between aligned tubes to observe the transmission. The quantitative results for reflection and transmission coefficients obtained are somewhat inaccurate, but qualitatively they are valid. Diffraction and background noise are the main problems encountered.

Objective: Reflection of a sound wave.

Method: The sound wave from the generator tube is directed at a fixed angle toward a hard flat surface. The receiver tube is adjusted with respect to angle and the intensity of the reflected sound measured as a function of the reflection angle.

What is the angle of the main reflected wave? Compare your observations with the results reported in the short concept movie on Bragg reflection from a regular structure.

Observe the reflection of sound from a regular footbridge railing using the observations you have made on reflection.

EXPERIMENTS ON INTERFERENCE
BETWEEN TWO COHERENT SOURCES

Interference occurs when two or more wave trains meet if

1) they are of the same frequency - otherwise they will not give a simple singly periodic disturbance when added together.

2) they are coherent - this means, not only must they have the same frequency, but there must be a constant phase relationship between them. For example, A and B are coherent, but C and D are not.

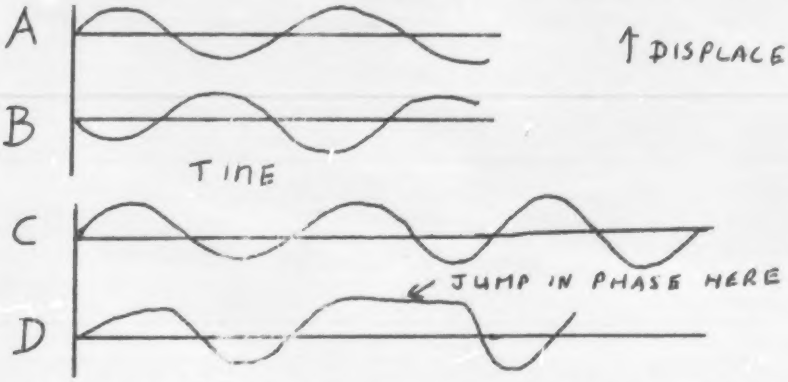


Fig 1

A. Method - To demonstrate the principle involved, set up the ripple tank, and examine the pattern produced with a single small round dipper. Cylindrical waves are produced and spread out from the source. They may be examined through the hand held stroboscope, which, if rotated at the correct speed, provides an aperture to look through when successive waves are in the same relative position, bringing the wave pattern to rest. If we replace the single dipper by a double one, we get two cylindrical patterns superposed. Two such sets of concentric rings give a Moiré pattern, as shown in the figure, which can be clearly seen in the ripple tank. Note, since the dippers are attached to the same bar, they not only have the same frequency, but are also coherent, i. e. always bear the same phase relationship.

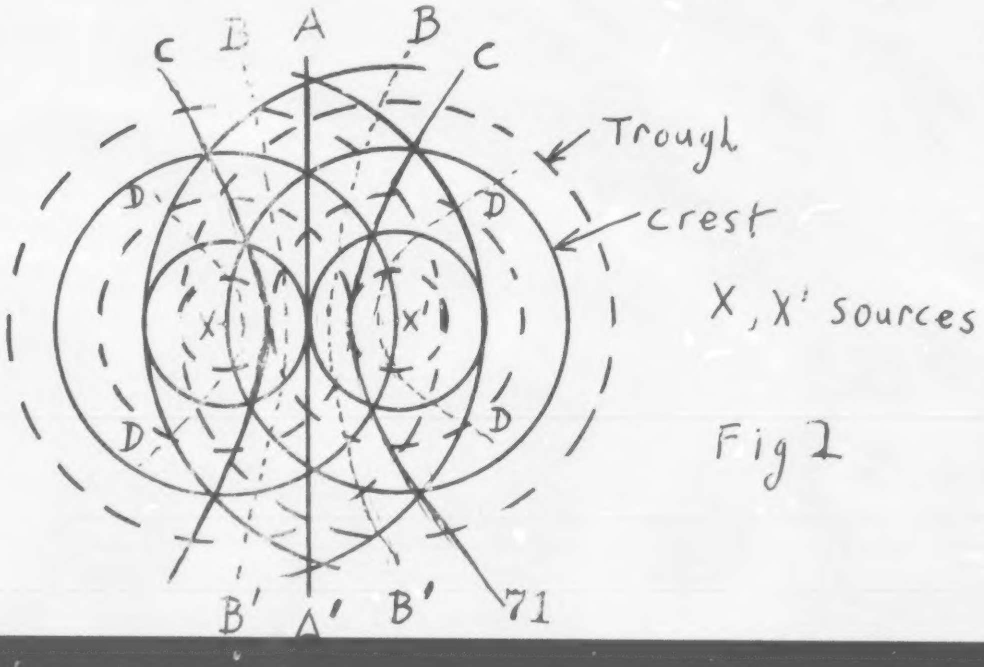
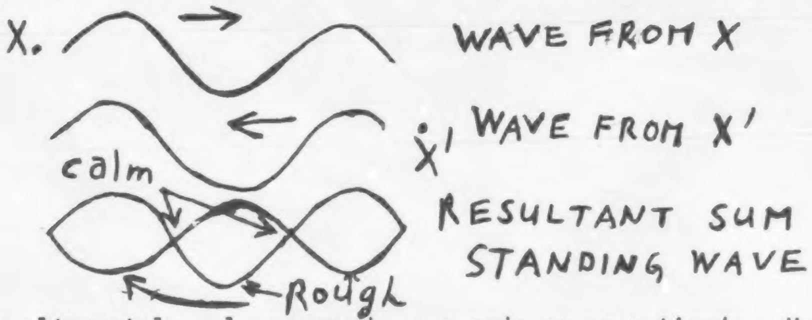


Fig 2

Each point on the line of the pattern labeled A in the figure is equally distant from the two centers -- so along this line crests and troughs coming from both sources reinforce, and produce crests and troughs twice as big as from one source. If we move away from this line to line B, here the crest from one source arrives at the same time as the trough from the other, cancelling, so the surface of the ripple tank is undisturbed or flat along line B. The distance from a point on line C to one source, however, is just one wavelength more than to the other. This means troughs and crests again arrive in phase from the two sources, and we get big maxima and minima. For D, we have calm again, and so on. If we examine the surface along line X X', we see it is a standing wave such as we had on a stretched string.

Fig 3



The surface is alternately calm or rough -- a node or an antinode. However, if we look along the direction A A' at right angles to this, we see we have a traveling wave, as the crests and troughs move away from the sources. So this pattern of interference on a plane is like the two cases of traveling and standing waves on a string. If the waves travel in opposite directions, interference gives us standing waves, as at X X', but if they travel in the same direction and are in phase, we get a large traveling wave, A A'. If however, they are out of phase, we get nothing, as B B'.

Photograph the pattern. Measure the distance directly between the dippers, count the number of waves between the two dippers and calculate the wavelength (don't forget these are standing waves). Measure the frequency of the rippling device using a stop watch. Calculate ripple speed from the equation speed = frequency x wavelength.

B) If, instead of ripples, we use sound waves, we can still see the interference pattern. Mount two small transducers (crystals or mylar foil that can act both as microphones & loudspeakers) with a separation distance D of 10 to 20 cm on a wooden beam. Attach them, in phase, to the oscillator, set at a frequency of 40,000 cps where the transducers are resonant. Place a third small transducer a distance L, equal to 30 cm, away, and move it slowly crosswise, as shown in Fig. 4 noting the maxima & minima, with the microphone connected to the oscilloscope. Map the maxima and minima on a sheet of paper, and a pattern appears closely resembling that of the ripples with two sources. Now, we would like to find the wavelength of the sound from the interference pattern. Pick a suitably large distance from the source, and measure the distance from the center maximum E to the next maximum out, F as shown. The distance from A and B to E is the same, so the waves are in phase. Now, for waves from both sources to be in phase at F also, the distance AF must be larger than the distance BF by just one wavelength, i.e. the distance AG is equal to λ. The angle FH E = θ is the same as the angle ABG, because BG is perpendicular to HF. This means $\frac{AG}{AB} = \frac{FE}{EH} = \theta$ or

$$\frac{\lambda}{D} = \frac{\ell}{L}$$

Hence find λ from $\lambda = \frac{\ell D}{L}$

Use several estimates for ℓ, and use different values for L. Hence calculate λ, and its error, and then calculate the velocity of sound from the known oscillation frequency from speed = frequency X wavelength, as for the ripples.

3) Lastly, we would like to perform exactly the same experiment with light. For this purpose a HeNe laser is used. Two slits with a known separation found by a microscope are held in front of the laser beam with a rubber band and a white screen placed 5 meters away. The two slits act as coherent sources -- there is always an exact phase relationship between the two slit sources, because the light from both originates from one source -- the laser. For light waves, two coherent sources can only be obtained by splitting the light from one source. In sound, the two transducers were also coupled to the same source, but it would have been possible to use two separate oscillators -- not so in light, because it is difficult in practice to keep the phase exactly constant. On the screen the maximum and minimum of the interference pattern appear as a series of light and dark bands, and, using the same nomenclature as for sound, so that ℓ is the distance between bright lines on the screen, we may use the same formula to determine the wavelength,

$$\lambda = \frac{D\ell}{L}$$

Hence calculate the wavelength of the laser light, and give the error.

Conclusion

The phenomenon of interference between waves emanating from two coherent sources is a universal phenomena of wave motion. We have seen how it occurs in ripples, sound, and light waves, and can be used to measure the wavelength.

Equipment - Ripple tank - McCallister
Welch
etc.
Transducers - Available from most electronics stores - used to change channel on television receivers

References:

Backus "Acoustical Foundations of Music" p. 68.
Wood "Physical Optics" p. 160.

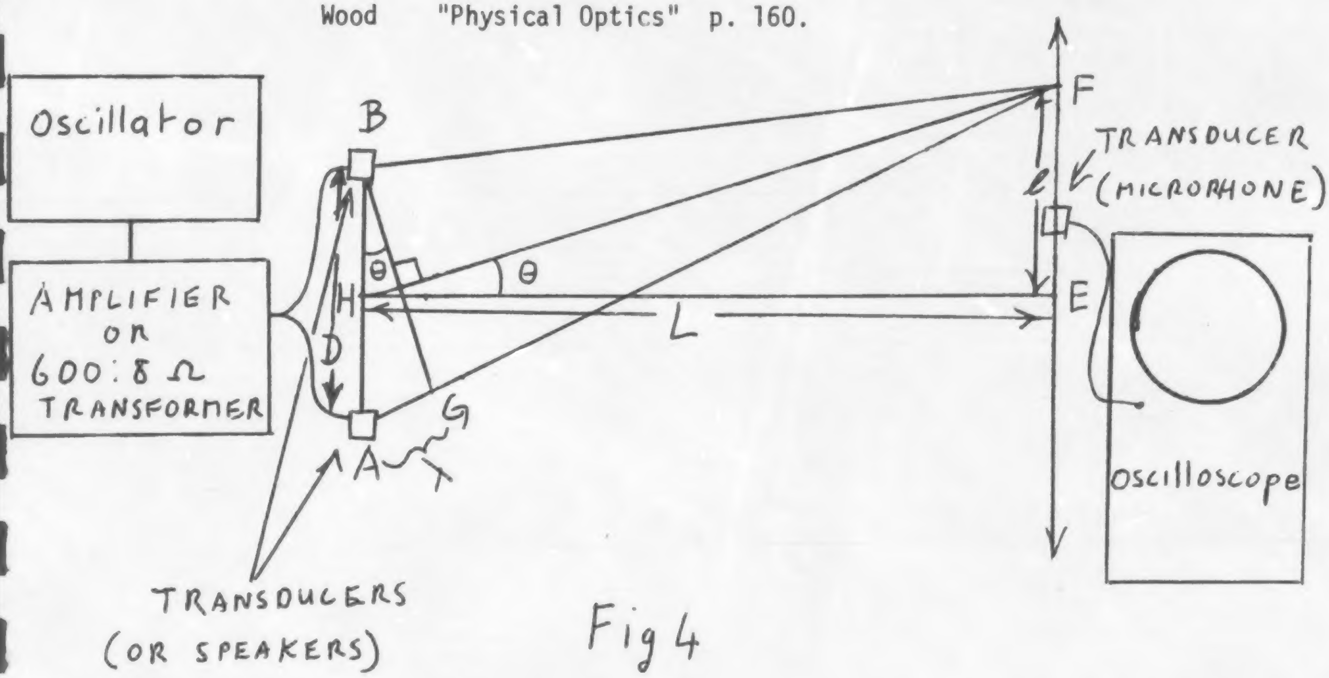
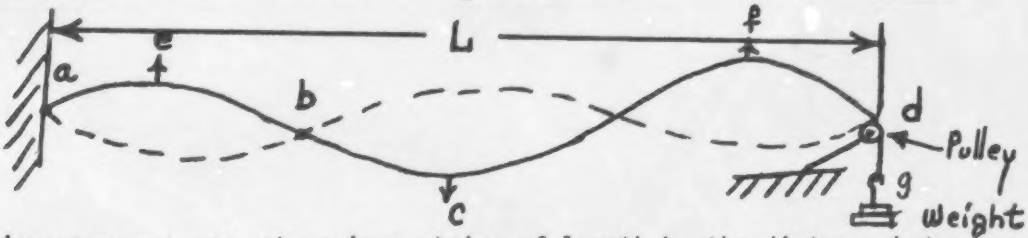

Oscillator
AMPLIFIER
OR
600:8 Ω
TRANSFORMER
B
H
D
A
G
θ
θ
L
F
ℓ
E
TRANSDUCER
(MICROPHONE)
Oscilloscope
TRANSDUCERS
(OR SPEAKERS)

Fig 4

VIBRATING STRINGS

Whenever a guitar or violin string is stretched tighter or made shorter, the pitch at which it sounds when bowed or plucked is heightened, i.e., the vibration frequency increases. It is the purpose of this experiment to demonstrate the various physical parameters on which tuning depends.

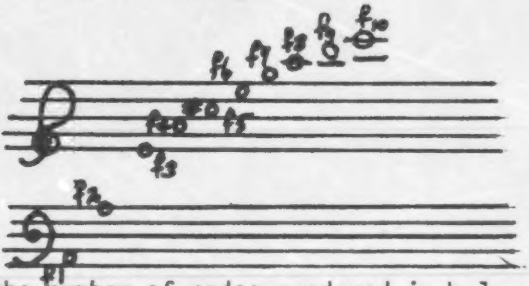
Theory: Stationary waves are set up in a stretched string when two equal trains of waves traveling in opposite directions are impressed upon it. Certain points called nodes such as a or b in the figure, are never displaced from their rest positions. All other points on the string are in constant vibration, the maximum vibration occurring at the loops midway between the nodes such as points c or e on the string. The portion of the string between the nodes is called a segment.



If stationary waves are set up in a string of length L, the distance between a and d, a node is formed at each end, a and d, and also at other points such as b. Do not include the length of string d to g in L since it is not vibrating. The term wavelength is defined as the distance along a wave between 2 adjacent points that are vibrating in phase, i.e., traveling in the same direction and with the same speed. Two such points would be e and f so that the distance between them would be one wavelength, λ. Likewise, the distance between b and d is also λ. The distance between e and c is $1/2\ \lambda$ since e and c are vibrating 180^0 out of phase. The corresponding length of $1/2\ \lambda$ would be the length of a segment, a to b. In the length L, there are an integer number of segments, or an integer number of half wavelengths. Call this integer, k. So, k may be 1, 2, 3, etc. In the figure, k=3 since 3 segments are represented. The tone sounded by the string in this mode is called the third harmonic; i.e., an integer multiple of the fundamental, f_1. Thus $f_3=3f_1$, $f_4=4f_1$.

$$L = k(1/2\lambda)$$

The familiar harmonic series is thus generated.



$A_2 = f_1 = 110\text{Hz}$
$A_5 = f_8 = 8f_1 = 880\text{Hz}$

In the string itself, the number of nodes produced is k-1. Thus A_5 would contain 7 nodes.

The speed of the wave, V is given by the product of the frequency and the wavelength, $f\lambda$. Elimination of the wavelength yields

$$V = (2fL)/k$$

The speed of the wave depends uniquely on the tension in the string, F, and the linear density, μ. One way to determine μ would be to measure the mass of the string between points a and d and divide this by L. Thus, convenient units for μ might be gm/cm. The tension F, is given by the weight which hangs on the end of the string. F will be the sum of the weights of both the slotted weights and the weight holder. Be certain to use the correct units in your equation, e.g., dynes instead of gm .

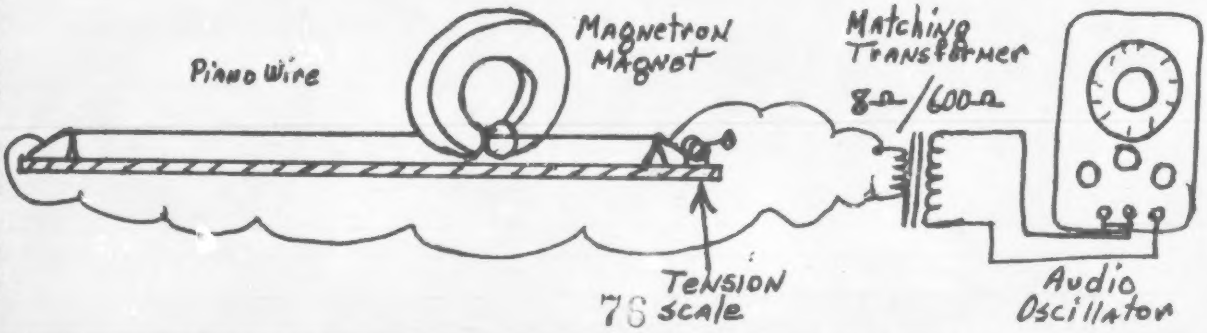
$$V = \sqrt{F/\mu}$$

Eliminate V and solve for .

$$\mu = F(k/(2fL))^2$$

Apparatus: In order to sustain stationary wave patterns along the length of the string, 3 systems of energy coupling will be described.

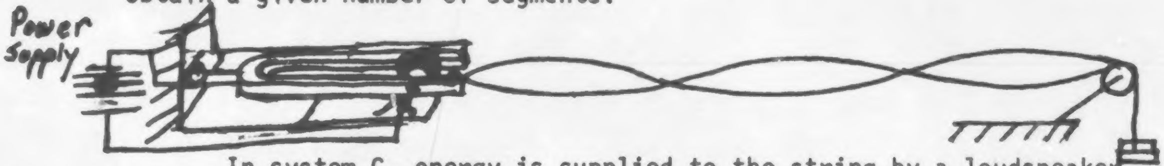
System A consists of an alternating current-carrying wire positioned in an externally applied magnetic field. If an audio oscillator signal drives a power amplifier, the output may supply several amperes to the terminals of a monochord. The monochord consists of a board on which two bridges are mounted to support a steel wire. A spring scale can be placed between the wire terminus and the bridge.



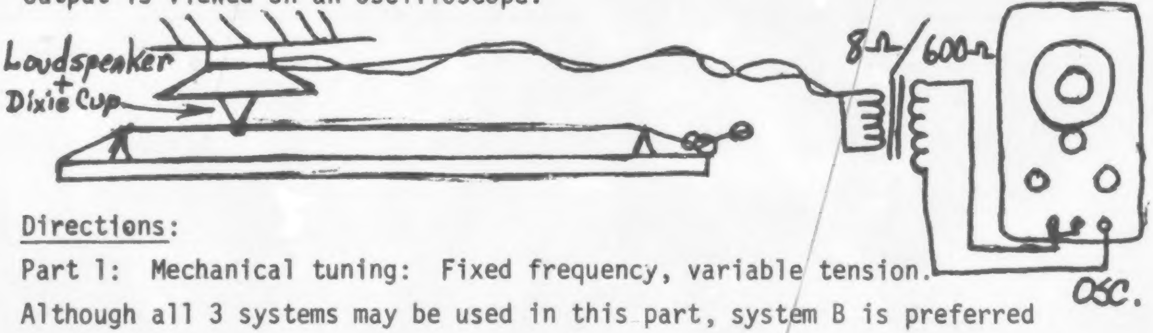
to measure the wire tension. To improve coupling, the field position should be adjusted away from nodes. Care should be taken not to overheat the wire.

System B is the standard electrically driven tuning fork device as produced by Cenco. Because of the mechanical operation, overtones of the source are suppressed and a purity of wave character obtained.

The string can be passed over a pulley and attached to a weight holder for tension measurements accurate to 5 gm. The experimenter increases or decreases the tension by pushing or pulling on the weight pan and noting if an increase or decrease in attached weight is required to obtain a given number of segments.



In system C, energy is supplied to the string by a loudspeaker driven by an audio oscillator. The coupling device between the speaker and string is the inverted cone part of a Dixie-cup cut to fit inside the speaker and voice coil with silicone plastic (bathtub calking compound). A speaker should be chosen such that the height of the Dixie cup should exceed the depth of the speaker in order to have string contact. Contact is maintained with the string via a thin cut in the point of the cup-cone. If an identical "transducer" system, is placed at the other end of the string, this second speaker may be used as a resonance detector. The output is viewed on an oscilloscope.



Directions:

Part 1: Mechanical tuning: Fixed frequency, variable tension.

Although all 3 systems may be used in this part, system B is preferred for accurate tension measurements. It seems superior to the spring scale and tuning key. Vary the tension and record its value for maximum displacement amplitude of from 1 to 5 segments. Measure the length, L of the vibrating portion of the string, e.g., from the tuning fork to the pulley.

Note and record the driving frequency, f. Calculate μ for each K. Determine the average and compare it with the value of μ directly obtained, i.e., measure the weight of the string and find its ratio to the entire length. To see the functional relationship of the tension with the number of segments, plot a graph of $1/\sqrt{F}$ as ordinate versus K as abscissa.

Part 2: Electrical tuning: Fixed tension, variable frequency.

System B is not to be considered for this part since the tuning fork tension is constant. The tuning is accomplished electrically by varying the audio oscillator frequency. Determine the fundamental frequency (first harmonic) at which a single segment occurs (K=1). Increase the frequency by about an octave to locate the resonance of a single node (K=2). This may not be exactly twice the frequency due to stiffness effects in the string. This error is reduced at high tension. Continue for as many modes as are observable. Plot a graph of frequency as a function of number of segments. If the tension is available, compute the linear density, μ and also the wavespeed, $\overline{V}$. Compare this to the product of wavelength times frequency, λf by obtaining the product of the string length L with the second harmonic frequency, f_2.

Part 3: Fixed tension, variable length.

This establishes the relationship between higher pitch and shorter string length. Start with a low frequency in systems A or C and tune the frequency to the fundamental mode in a long string. Reduce the string length and retune to the fundamental. Make 5 determinations and plot a graph of the reciprocal of the fundamental versus string length, i.e. $1/f$ versus L.

Part 4: Variation of Linear Density

Repeat the above experiments using different media. For a fixed tension and length, obtain μ for steel wires of varying diameters, nylon, cord, and thread strings. Determine the fundamental frequency of each. Which type of material should be used for high notes, which for low?

Part 5: Determination of resonance response.

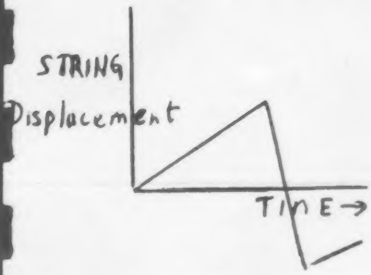
In system C, measure the amplitude of the fundamental A_o, i.e., the maximum displacement of the wire of a monochord. Note the fundamental frequency f_o. Decrease the frequency below resonance to f_1 where the amplitude A_1 is approximately p.707A_o. Record f_1. Now tune to f_2 above resonance where $A_2=A_1=0.707A_o$.

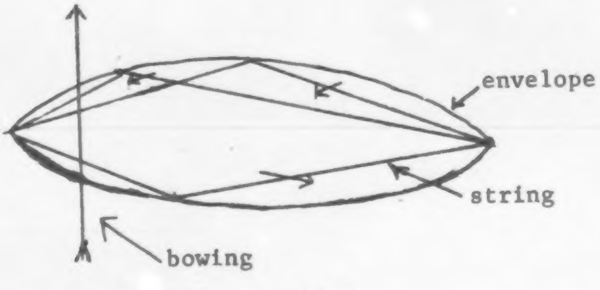
The system Q is defined by $f/(f_2-f_1)$. Determine Q_m. If an oscilloscope is available or AC voltmeter as a second choice, read the output voltage at resonance, V_o. Determine f_1 at $V_1=0.707V_o$ and f_2 at $V_2=V_1$. Find Q_E, from the electrical determination and compare it to Q_m.

Part 6: Stroboscopic Effects.

Tune to a string resonance with an audio oscillator. In a darkroom, adjust the strobe frequency to match the oscillator frequency. Strobe light on a white string against a black background is impressive. If a Type 1531-A Strobotac is used, the oscillator can trigger the strobe automatically and produce precisely stationary effects such as a string "hanging" upward in a bowed curve. Be sure to multiply the strobotac reading by 60 since the scale is RPM rather than Hz. One advantage of the automatic triggering is the strobe frequency of the fundamental is immediately found. With manual strobe tuning, it is possible to "come across" a sub-multiple, such as 1/2 the fundamental frequency in which the half sine wave pattern would be correct although the string is illuminated only on alternate vibrations. However, after the correct frequency is established regardless of harmonic, a flashing frequency just off the string frequency gives the more desirable effect of a slowly varying string. Tune the strobe frequency to twice that of the string. With two flashes per vibration, two strings appear. Adjust the strobe frequency, "phase" until the strings appear at their amplitude. Triple the frequency and view three strings. Try half this frequency, i.e. 1 1/2 the string frequency. Describe and explain what is seen.

If a violin is available, bow it. Then view under strobe light the resulting complex waveform. Sketch it. Read the article in the January 1974, issue of Scientific American (p. 87) on "The Physics of the Bowed String" by John C. Schelleng. The waveform is formed by the intersection of two straight lines. Note the circulation of this intersection with the strobe.





A STUDENT PERCEPTION OF VIBRATO

When a vibrato is sounded, the ear of the listener hears the top pitch of this varying note. When a violinist plays his note, his fingers vibrate to lengthen the strings and thus provide lower frequencies under the note. Do musicians agree with this? Some folks do.

When a vibrato is sounded, the ear of the listener responds to the average pitch played between the top and bottom tones sounded. Do music students feel this is correct? Some folks do.

When a vibrato is sounded, the ear of the listener responds to any note needed or subjectively selected by that listener from the vibrato range. Thus a note may be picked in the interval to complete a musical phrase or melody or to coincide with a note from another musical instrument. Do you consider this correct? Some folks do.

A pertinent comment on this topic is made in an article by W. Dixon Ward entitled "Musical Perception", p. 423 contained in "Foundations of Modern Auditory Theory", Vol. 1, Jerry V. Tobias, ed. Academic Press, 1970. The article states: "That is, although the judged pitch of a tone with vibrato heard in isolation may turn out to be the same as that of a steady tone at the mean frequency level, it is possible that an interval will be acceptable if the varying tone merely includes the frequency that the listener considers to be the correct one for a particular interval. This notion was expressed by Kock in 1936: 'Thus a note originally off pitch may be made acceptable by imparting a frequency vibrato to it, provided the intended pitch is included in the vibrato interval and provided the vibrato interval is not too wide to be objectionable.'"

It is the purpose of this experiment to investigate how the ear responds to the vibrato and report sample results for a guideline. The test subjects were 14 physicists attending a Physics in the Arts Workshop sponsored by the National Science Foundation at the University of S.C., the summer of 1974. In short, the subject matched the pitch of an audio oscillator to the remembered pitch of a vibrato from a tape recorder. The tape was prepared from the output of a voltage control oscillator. Essentially, the subjects attempted to match the pitch of a frequency modulated wave by memory only. If they were to hear the tape vibrato along with the audio oscillator sound there would be a tendency to match beats at the upper or lower frequencies rather than obtain a pitch perception. A simple toggle switch provided the means to listen to one or the other.

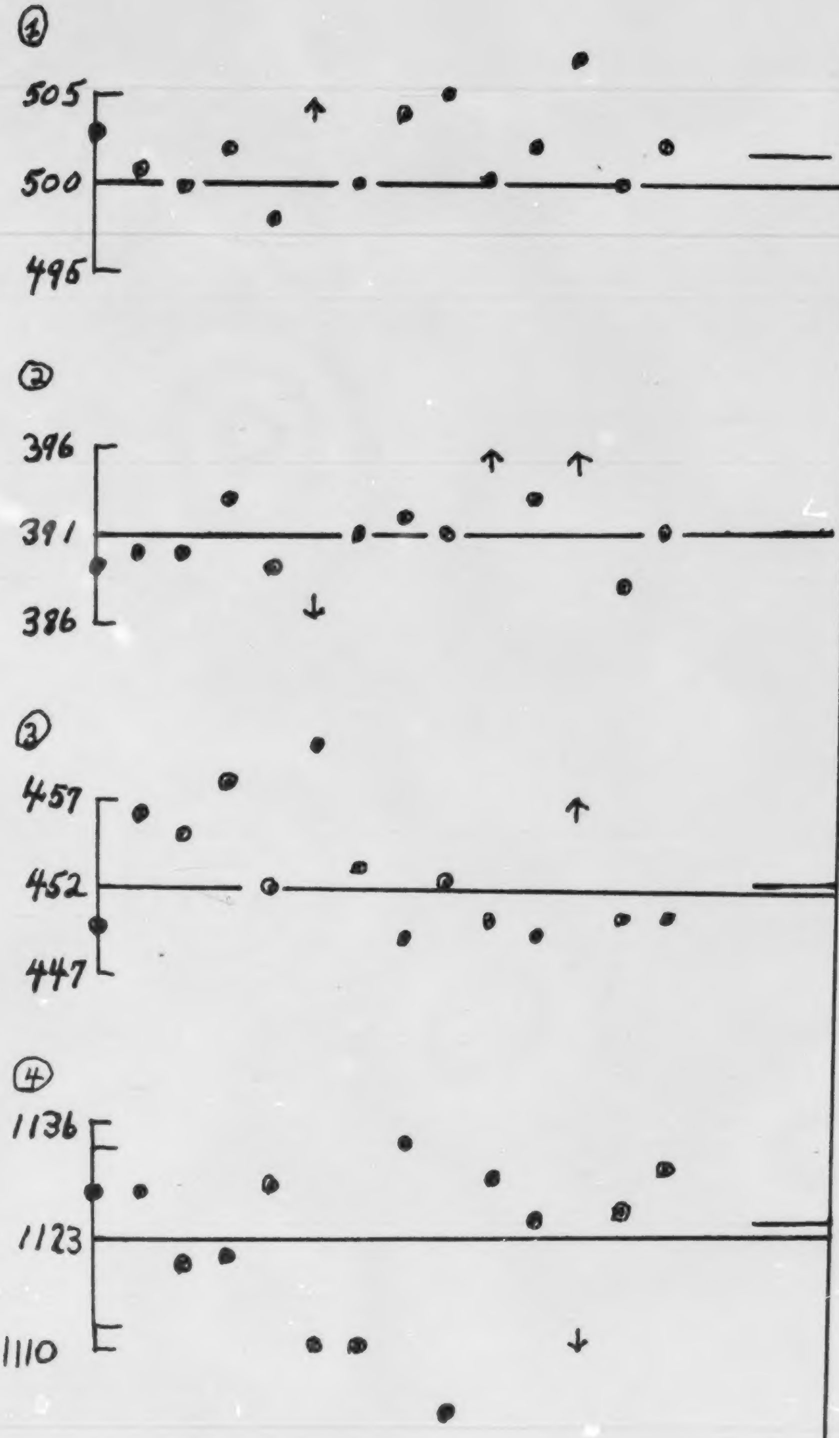
To prepare the tape, test parameters must be selected. Musicians feel that vibrato rate, or the modulation frequency along with vibrato depth (quarter tone, half tone) are mainly responsible for controlling the warmth and emotion that their instruments produce. In the South Carolina tape, vibrato rates of 3, 5, and 8 Hz. were used with a depth of from 35 to 91 cents. Two other parameters included in the test were the basic frequency and the type of modulating signal generated. Sine and triangular waves were selected, the latter so chosen that the rate of frequency sweeping was the same in any differential frequency range. Most of the tests were run within 15% of middle A. High and low frequencies increased subject difficulty.

The upper and lower frequencies of the vibrato can be obtained by connecting a frequency counter to an audio oscillator. A Lissajous figure is formed on an oscilloscope from the voltage controlled oscillator and the audio oscillator. When the Lissajous figure becomes momentarily stationary (and does not move in reverse), the frequency meter reads one of the vibrato pitch extremes. As a check, the frequency meter can read the output of the VCO and integrate over a time span much greater than the vibrato period. Thus the average frequency is obtained.

With the 4 parameters precisely identified, the test was designed to give a variety of combinations. Test I was a monotone to check if the subject might be tone deaf. It also served as a control to see how accurately a subject could match the correct answer, provided there was one.

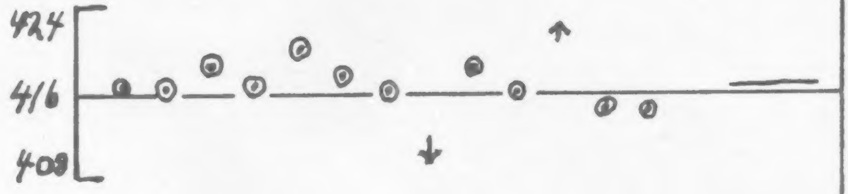
Test results indicate that although an individual might select a pitch any where within the full depth of the vibrato range the entire group average was quite close to the average provided by the frequency meter. The 9th and 12th test subjects (as indicated by counting the dots from left to right) have arrows indicating that they selected pitches outside the vibrato range. Perhaps there was a matching of chords. The last 2 test subjects were singers and seemed to come consistently close to the average. Can you determine the group to which your ear responds? Some folks do.

Vibrato Test -14 Subjects



	vibrato Frequency (HZ)	Depth (cents)	Signal
①	0	0	~
②	3	45	~
③	5	35	~
④	5	40	~

⑤



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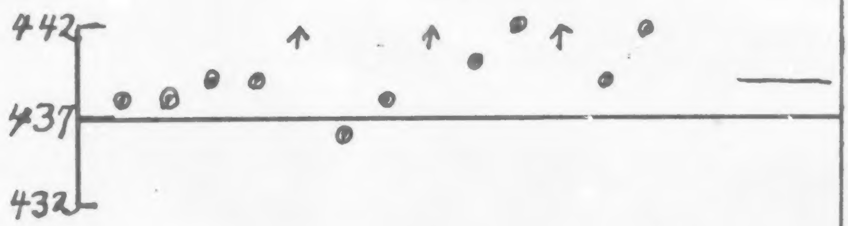

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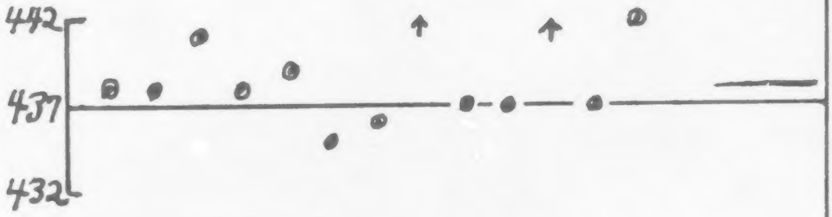

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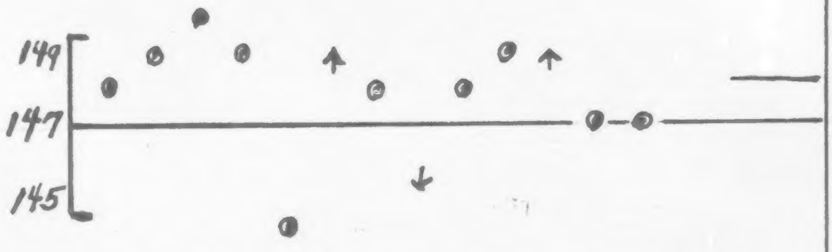
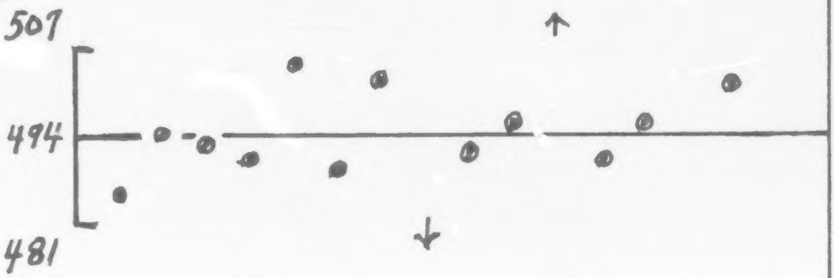


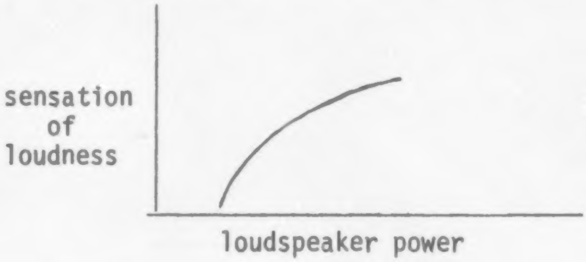
⑨



5	63	N
5	40	N
8	40	N
5	48	N
5	91	N

LOUDNESS LEVELS

From your reading in the text you will know that your ear does not perceive the intensities of sounds in a linear way. If the power to a loudspeaker is doubled the sound you perceive will not seem twice as loud, instead it will seem much less than twice as loud. A graph of your sensations of loudness plotted against the power to the loudspeaker would look something like this -



Because this graph is not a straight line we say that the sensation of loudness is a non-linear function of the sound power. Sound levels are most frequently measured in decibels (dB), an arbitrary scale based on the logarithm function, which only crudely approximates the response of the ear. We convert other physical measurements to the decibel scale as follows:

Acoustic	Electrical
$L = 20 \log p/p_o$	$L = 20 \log v/v_o$
$L = 10 \log I/I_o$	$L = 10 \log P/P_o$

In these equations L is the sound intensity level in decibels and

p is the sound pressure in dynes/cm^2
I is the sound intensity in watts/cm^2
v is a voltage in volts
P is a power in watts

All those quantities with a subscript "o" are reference levels. For acoustic measurements the reference is usually the threshold of hearing. Therefore, I_o, for example, is usually 10^{-16} watts/cm^2.

In audio electronics the measurement of sound levels are made electronically. There is no such natural reference point as the threshold of hearing. In practice a variety of reference points may be used. For example, the amount of noise added by an amplifier to the musical signal might be quoted as, "Signal to noise ratio = 80 dB." This means that

$$80 = 20 \log \frac{v \text{ signals}}{v \text{ noise}}$$

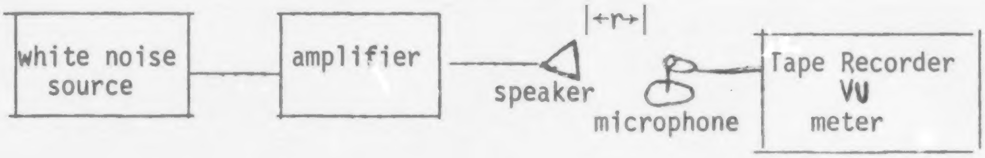
Comparing this formula with the above table you can see that the average noise voltage is taken as a reference. The above statement says that the average signal voltage is at 80 dB relative to the reference noise voltage.

Electronic readings of audio signal levels are made on a VU meter (volume units). On the next page you will find an accurate drawing of the face of a typical VU meter used in magnetic tape recording. There are two scales on this face the lower, smaller, scale reads voltage as a percentage of the optimum recording voltage. When the meter reads 100% then the signal to noise ratio will be as large as it can be without distortion due to excessive signal voltage. The upper scale reads decibels. Notice that 0 dB on the meter corresponds to the optimum recording voltage, 100%. Therefore, you know that the reference point for the VU meter decibel scale is a reference voltage equal to the optimum recording voltage.

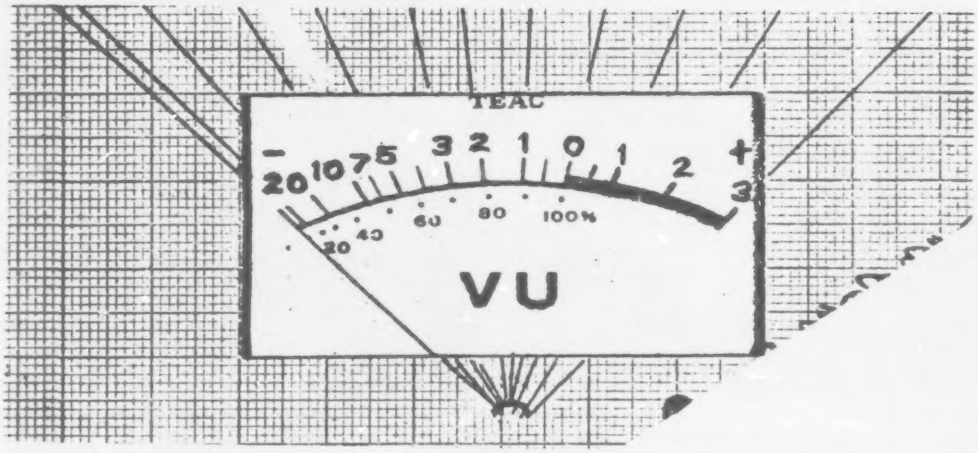
Let's check out the decibel scale of the VU meter by comparing it with the voltage scale in %. Our basis for comparison will be the angle Θ of needle deflection. Because of the mechanics of the meter movement itself, the voltage scale is not exactly linear with Θ. Use a protractor to measure the angle of meter deflection from the left hand edge of the scale for relative voltage readings given by the dots on the lower scale at 20, 30, 40, 50, 60, 70, 80, 90 and 100%. The table of your readings will be the function $v(\Theta)/v_0$, relative voltage as a function of angle. Now use a piece of graph paper to plot $20 \log [v(\Theta)/v_0]$ on the vertical axis against Θ on the horizontal axis. The curve that results from connecting the points should be the decibel curve. Finally use your protractor to measure the angles for the indicated readings of the decibel scale -20, -10, -7, -6, -5, -4, -3, -2, -1, -0.5, and 0 dB. Compare these readings with the curve you have drawn.

Experiment: To use the VU meter to demonstrate the inverse square law of sound intensity.

Apparatus:



23.3


TEAC
− 20 10 7 5 3 2 1 0 1 2 + 3
20 40 60 80 100%
VU

According to theory the intensity of sound at the microphone obeys the relation

$$I \propto r^{-2}$$

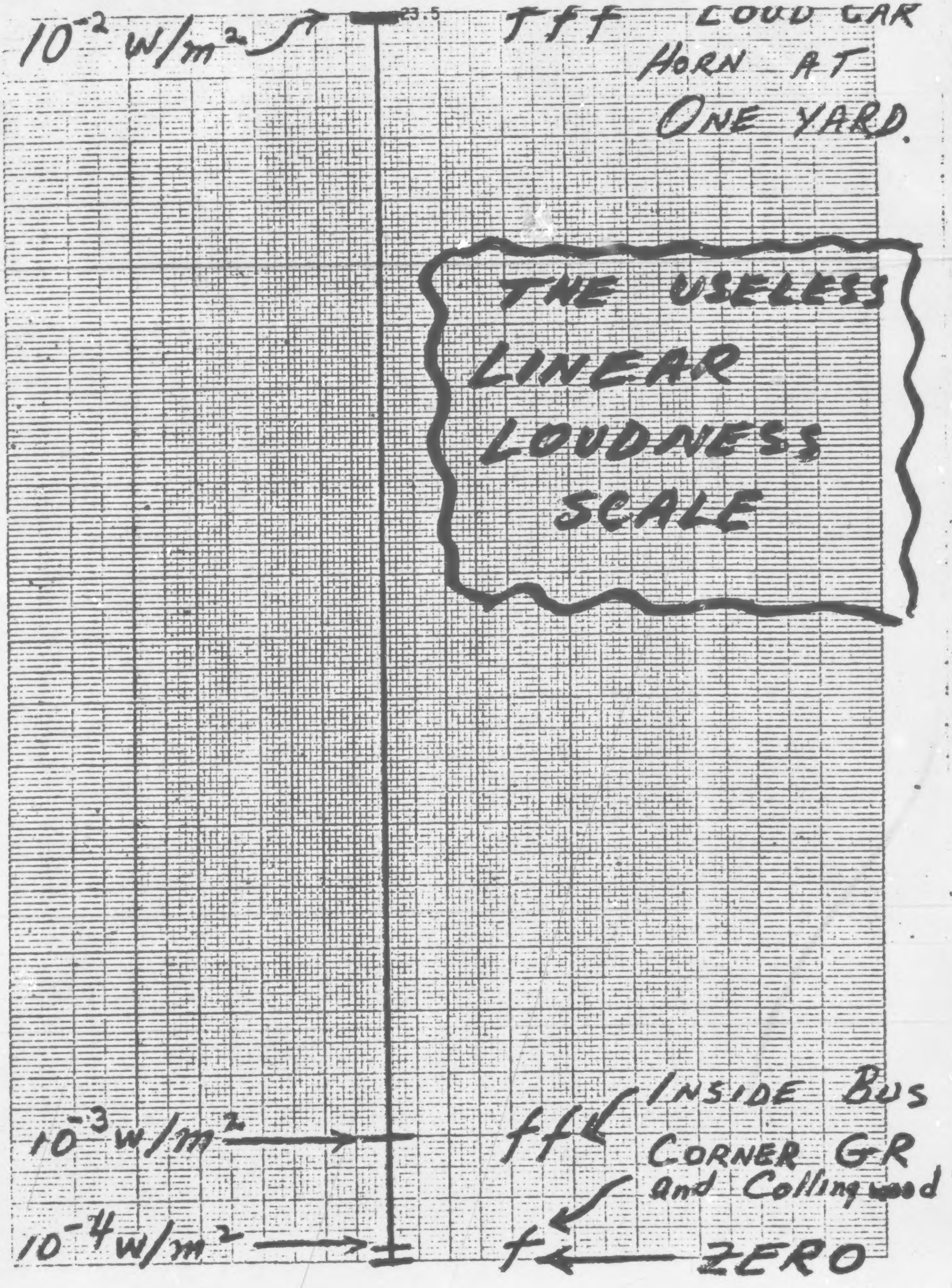
so long as r is not small compared to the dimension of the speaker. Suppose that r is doubled. By how many decibels do you expect the VU meter to drop?_________. Test your answer experimentally. To get the most accurate reading set the amplifier gain and tape recorder input level so that the VU meter reads 0 dB. This is then the reference intensity. Now double the distance between microphone and speaker and again make a reading.

Questions:

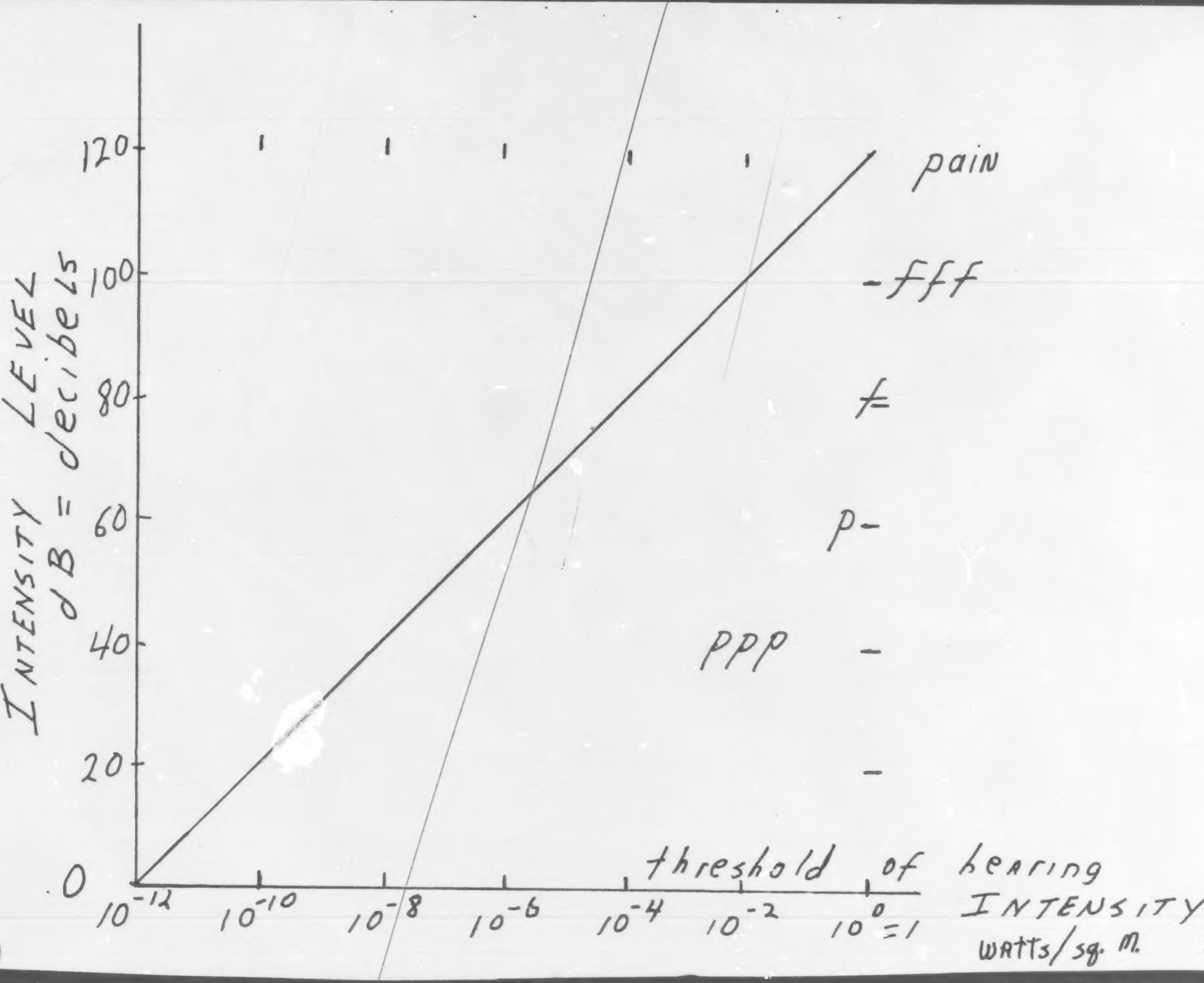
1. Why do we use a noise source rather than a pure tone for this measurement?
2. What are the effects of surrounding objects, such as tables, walls, floor, ceiling, on this experiment?

Equipment:

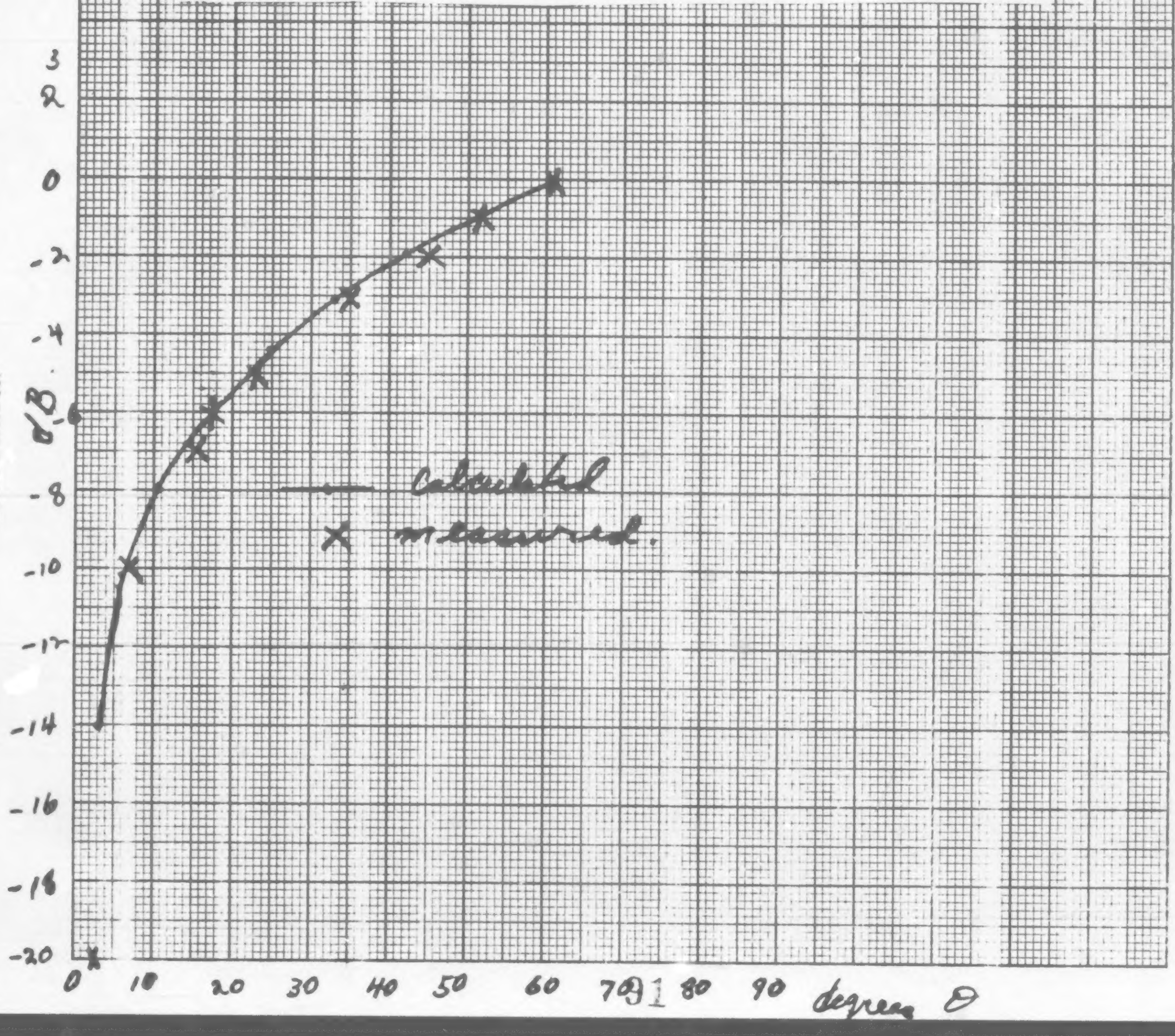
- Accurate drawings of VU meter face
- Protractor
- White Sound Source (an FM radio tuned to an unused channel will do)
- Amplifier
- Speaker
- Microphone
- VU Meter (a tape recorder with a VU meter will also provide the necessary microphone preamp.)



THE USELESS LINEAR LOUDNESS SCALE

INTENSITY LEVEL
dB = decibels
120
100
80
60
40
20
0
pain
fff
f
p
ppp
threshold of hearing
10^-12
10^-10
10^-8
10^-6
10^-4
10^-2
10^0 = 1
INTENSITY
watts/sq. m.

Calculated				Measured	
θ	J	logυ	20 log v	θ	db
3	.20	-.698	-14.0	2.5	-20
5.5	.30	-.523	-10.5	7	-10
10.5	.40	-.398	-8.0	15	-7
17.5	.50	-.300	-6.0	23	-5
25	.60	-.222	-4.4	35	-3
33	.70	-.155	-3.1	42.5	-2
42	.80	-.097	-1.9	51.5	-1
51.5	.90	-.046	-.92	61	0
61	1.00	0	0	71	+1
				82	+2
				94	+3



THE VIRTUAL PROFESSOR

One problem with demonstrations of transverse vibrations of springs is the explanation of the phase reversal of a pulse that is reflected from a fixed end mount of the spring. Since the fixed end mount cannot move,the usual explanation involves superimposing two waveforms of identical shape but opposite polarity and propagation vector passing through the fixed mount. Since the sum of these two waveforms will be zero and independent of time at the fixed mount we say this leaves the mount unmoved. But what is the source of the wave with reversed polarity and propagation vector?

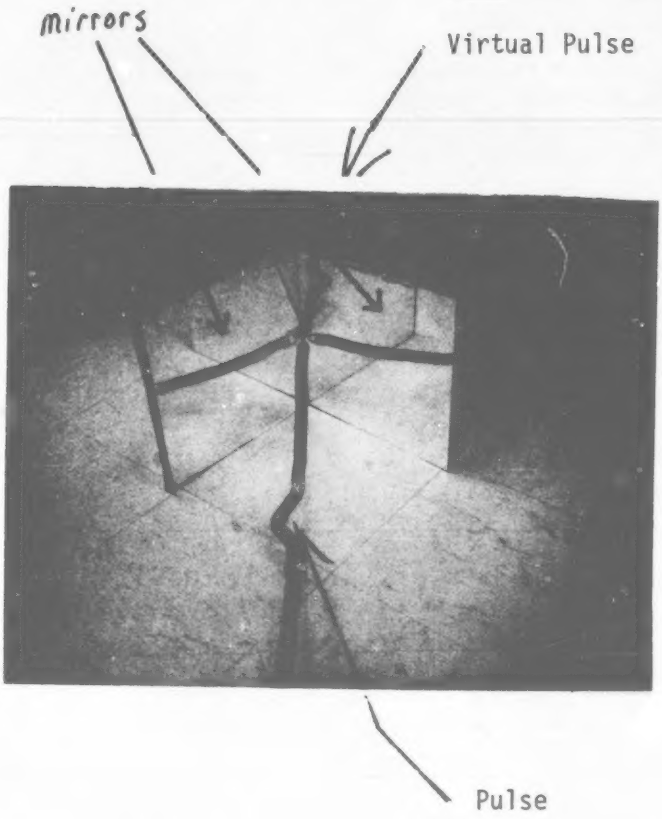
A simple mathematical trick with Fourier analysis creates zones that are familiar in crystal physics and other wave mechanical treatments. If we have a standing wave system we can divide the space wave. The zones will then be identical as far as the characteristics of the wave motion are concerned. We can say one zone alone is necessary to describe totally the original whole space if we just duplicate this zone to fill the space. A one dimensional zone would thus be thought of as having waves entering from the left and a second set of waves entering from the right with such an amplitude and phase as to cancel at the boundaries of the zones. The whole standing wave is thus described in the zone by these two sets of traveling waves.

In like manner to the zone we can envision the fixed wall mount of the spring as the boundary of a zone in which we are found. The traveling pulse moving toward the wall will be balanced by a pulse coming from the next zone (Virtually behind the wall) through the mount into our zone.

The pulse coming from the virtual zone will have opposite polarity and propagation vector compared to the one created in our zone.

As an approach to seeing into the zone behind the wall we place mirrors on the wall around the mount and the student sees the reflected image behave just as a "Virtual Professor" would. The symmetries are not exactly correct, however, since the pseudovector relationships of reflection of transverse displacement do not give an opposite polarity to the virtual pulse approaching the mount from the virtual zone. Both pulses appear to have the same polarity. The timing is obviously perfect however, in that the real pulse and the virtual pulse arrive at the wall mount simultaneously and proceed "through" the wall with a polarity reversal. The impression of passing through the wall is overwhelming, however, and emphasizes the effect stressed by the Fourier analysis.

The possibility of using a corner of mirrors was considered and a cursory experiment indicates that a very large mirror corner would be required due to the restrictions of the angular view generated by the corner reflection. Theoretically, the corner would give the proper polarity to the virtual pulse.


mirrors
Virtual Pulse
Pulse

Pulse and its image, the "virtual" pulse.

Practical Hint - It is best to use a "slinky" on the floor - the wave travels more slowly, and the polarity of pulse can be seen.

Reverberation Spring

Modern recording techniques leave as little as possible to chance. As the orchestra or band plays individual microphones pick up the sound of each section or each instrument (tight mikeing) and send those signals to individual tracks of a tape recorder. In making a rock recording there may be as many as 5 microphones and tape channels on the drums alone. Tape recorders with 16, 24, and 32 separate tracks are the industry standard for recording sessions. These separate tracks are then studied in detail, electronically modified or perhaps individually redone before they are mixed down to make the master tape. Because of the tight mikeing there is little reverberation or echo from the recording studio present on the tape. However, we are normally used to hearing music played in some enclosure or another and music without the normal reverberation of a room typically sounds dry and lifeless. This is particularly true of romantic works such as the symphonies of Beethoven. Therefore, reverberation must be added artificially.

The simplest and cheapest way to introduce artificial reverberation during mixing is to use a mechanical delay line or reverberation spring. Reverberation springs are used outside the mixing studio as well. They are used in Moog and Arp synthesizers to add realism to the electronic sound. Inexpensive reverberation springs can be added to automobile radios to provide a sense of depth to the sound.

The reverberation spring provides a relatively realistic simulation of reverberation in tha t its output includes several well defined echos followed by a jumble of sound with an exponential decay. The reverberation spring consists of a spring about 10 cm long with electromechanical transducers at each end. One of the transducers receives an electrical signal from an amplifier and converts the signal into mechanical vibrations of the spring. The mechanical vibration propagates down the spring until it comes to the second (receiving) transducer where it generates an electrical signal. The vibration is also reflected at that end, propagates back up the spring until it is reflected by the end of the spring where it started. The vibration then propagates down the spring again to the receiving transducer where it creates a second electrical signal. This reflection process continues until the energy losses due to friction in the spring reduce the amplitude of the vibration to zero. Because the reverberation spring is supposed to simulate the effect of a large room the time interval between echos is long, say 25-30 milliseconds. But a vibrational pulse has only to go about 10 cm (or 20 cm for a round trip) to create the echos. Therefore the speed of sound on the spring is very slow - typically about 1/100th of the speed of sound in air. The vibrations of the spring itself may be of several kinds. They may be compressional in which the coils of the spring are alternately squeezed together and separated. They may be transverse in which the coils of the spring move perpendicular to the direction of propagation of the sound wave. Or, they may be torsional in which the sound waves move by twisting the spring. To some extent all of these three types of vibrations are created on the spring, but one of them will be coupled most strongly to the transducers and will be by far the predominant vibration. Which of the three types of vibration is predominant depends upon the particular spring and the type of transducer coupling.

One of the practical difficulties with the reverberation spring is that its frequency response is a) limited to something less than the entire audio spectrum and b) very ragged with many peaks and valleys. This poor response leads to a coloration of the sound passed by the spring, namely the imposition of a tonality which is characteristic of the spring itself and not of the music being transmitted. In practice this problem need not be fatal because a) the reverb signal is usually mixed in rather small proportions with the original audio signal and b) the normal effect of a reverberant room is to color the sound somewhat. Therefore, the spring reverb does not sound completely artificial.

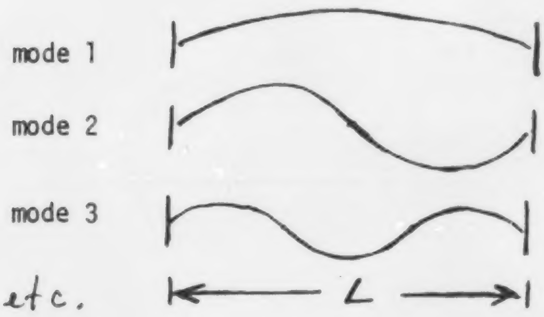
Experiment: This Experiment consists of several connected parts.

A. Determine the velocity of sound on the spring by introducing pulses and using an oscilloscope to measure the delay time for the pulse to propagate down the spring. You will be able to observe the first arrival of the pulse at the receiver as well as a number of subsequent echos.

B. Determine the resonance frequencies of the spring. It is the strong resonances of the spring that lead to coloration of the sound.

Theory of the resonances:

The spring is fundamentally a one-dimensional system with fixed boundaries. Therefore, the resonant modes of vibration are



$L = \frac{\lambda}{2}$; $f = (\frac{v}{2L})$

$L = 2(\frac{\lambda}{2})$; $f = 2\ (\frac{v}{2L})$

$L = 3(\frac{\lambda}{2})$; $f = 3\ (\frac{v}{2L})$

where L is the length of the spring

λ is the wavelength of the mode

f is the frequency of the mode

v is the speed of sound on the spring.

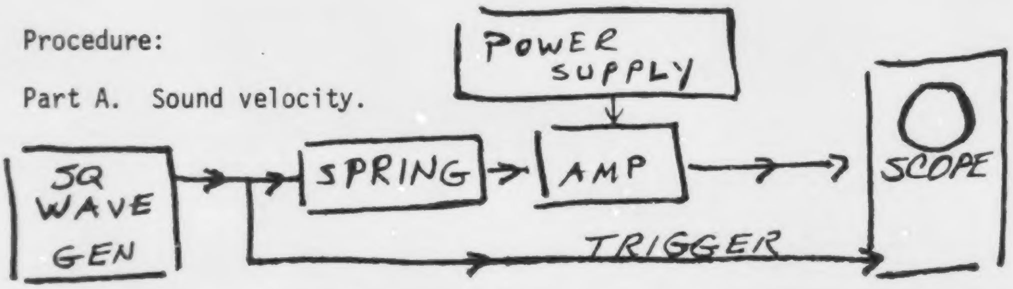
Notice that the resonances are equally spaced in frequency. The common frequency difference is (v/2L). You will be able to identify more than 30 resonances.

C. Visually identify several of the resonance-modes (or standing waves) of lowest frequency. i.e. the particular modes shown in the figure above.

D. Listen to some music played through the reverb spring.

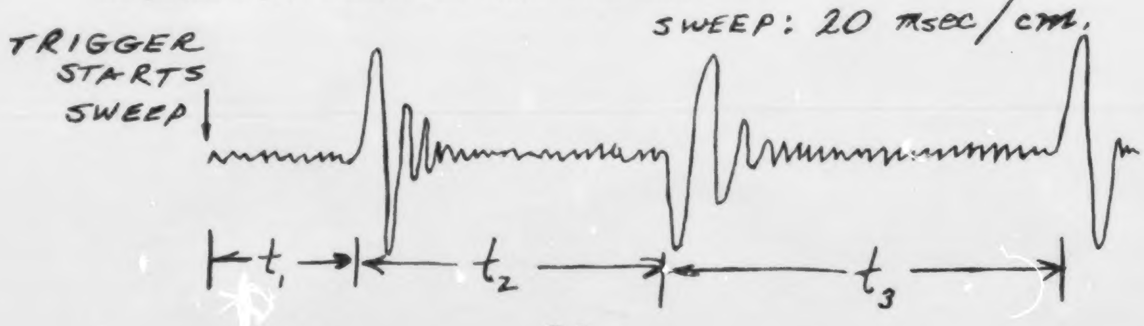
Procedure:

Part A. Sound velocity.



Set the square wave generator to produce two cycles per second. The reverberation spring will be sensitive only to the changes of the square wave which will appear as pulses. Adjust the output level of the square wave generator and the oscilloscope so that the output of the spring reverb can be seen on the oscilloscope screen. Now adjust the trigger setting so that the oscilloscope trace is initiated twice every second. Each successive pattern on the screen should appear identical. Although the reverb spring is transmitting 2 pulses for each cycle of the square wave, one positive the other negative, the oscilloscope sweep is triggered only on one of these pulses for each cycle.

The pattern you should see looks like this



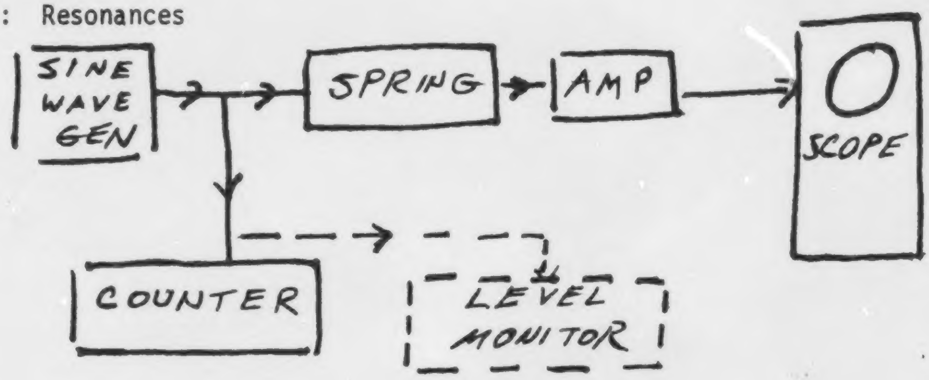
Using the calibration of the oscilloscope sweep measure the time t_1. This is the time required for the pulse starting at the transmitting transducer to reach the receiving transducer. Now measure t_2. This is the time for the first round trip and it should be twice as long as t_1, the time for the one-way trip. Now measure t_3, the time for the second round trip which should be equal to t_2.

Next measure, L, the length of the spring from the first coil to the last, neglecting the connecting loops at the ends of the spring. The speed of sound on the spring is equal to

$$v = L/t_1$$

You are now prepared to predict the common frequency differences for the resonances of part B. The common frequency difference is (v/2L), or, from the above formula, this common difference is ($1/2t_1$).

Part B: Resonances



The level monitor, which may be an a.c. voltmeter or the second trace of a dual trace oscilloscope, is useful if quantitative measurements of the height of the resonances are wanted. With this monitor one can adjust the oscillator output to be constant independent of its frequency.

Tune the oscillator around the region of 400 Hz where there are typically the strongest resonances. Adjust the oscilloscope input and the sine wave generator output so that the strongest resonance just fills the scope screen. It is best to apply only a weak signal to the spring from the oscillator and to amplify the spring output by using the most sensitive setting of the oscilloscope input, so long as the output signal is not badly distorted by noise vibrations within the room which are picked up by the spring. To reduce this noise it may be helpful to place the reverb spring unit on a piece of rubber foam.

Now make a table of the resonance frequencies. Tune the oscillator frequency carefully so that the resonance peaks appear as large as possible on the screen. The resonances are very sharp and therefore some time is required for them to develop. When searching for resonances you will have to tune slowly. Find the frequency differences between successive resonances and compare these differences with the expected value calculated in part A.

The low frequency resonances, at less than 100 Hz, are more difficult to excite than the higher ones. They are broader, not as strong, and often are not quite in tune with the expected frequencies. To observe these resonances may require increased output from the sine wave generator.

Part C: Observe some of the lowest frequency resonances by visually noting the nodes and antinodes of vibration. The lowest resonance has no nodes, the second resonance has one node etc, as shown in a previous figure.

By observing these low resonances you will be able to determine the type of vibration (compressional, transverse, or torsional) which is predominant in your spring. If your table of resonance frequencies is complete down to the lowest mode then the visual observation will provide a base from which you can assign the mode character of all the resonances you have seen.

Part D: As you listen to music through the spring listen for the coloration of the sound and for the decay time of the reverberation.

Equipment:

Part A: Any square wave generator capable of supplying 1 watt into an 8 Ω at 1 cycle per second. A step down transformer or an amplifier may be needed on this generator output.

Part B and C: A sine wave generator capable of supplying 1 watt into an 8 Ω load, 15-500 Hz.

A digital frequency counter.

Optional: Level monitor such as A.C. voltmeter.

Part D: A loudspeaker and source of music, e.g. A radio.

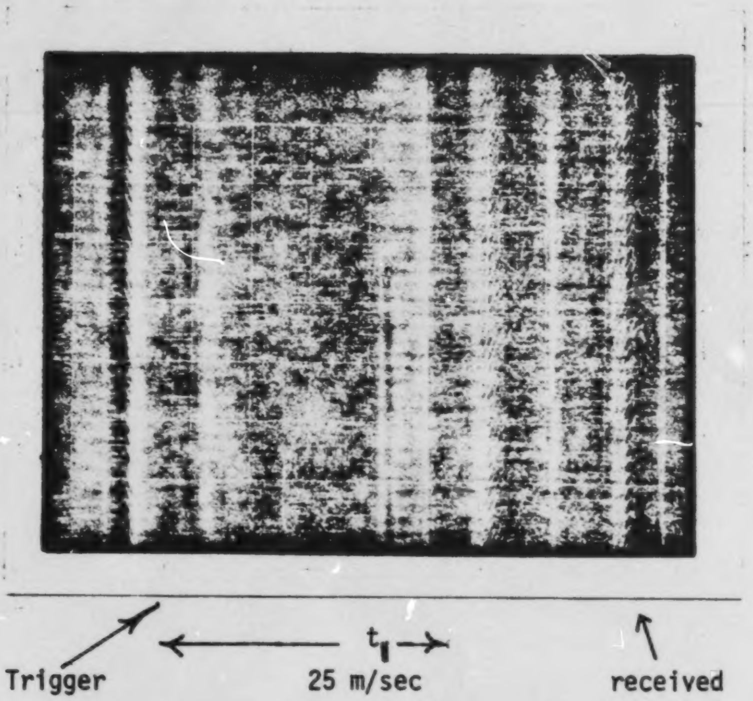
Generally: A spring reverb. These are most conveniently bought as reverberation units for rear speakers in an automobile under such brand names as "Stereo verb" or "Stereo magic." The reference to "stereo" is, of course, misleading. The reverberation does introduce a sense of depth into the sound as does stereophonic reproduction. These units sell for less than $15 and include a small power amplifier. With such a device no additional amplification after the spring is needed. However, these units do require a power supply, 12 v @ 0.5 amp. typ. When selecting a reverberation unit the rule to follow is to buy the cheapest one available. Inexpensive models will likely have the most pronounced resonances. Furthermore, some reverberation units are made with two parallel springs, both of them driven by and driving the same transducers. Such units may give better sound but are clearly unsuitable for this experiment. One is unlikely to find dual springs in the cheaper models of automobile reverbs.

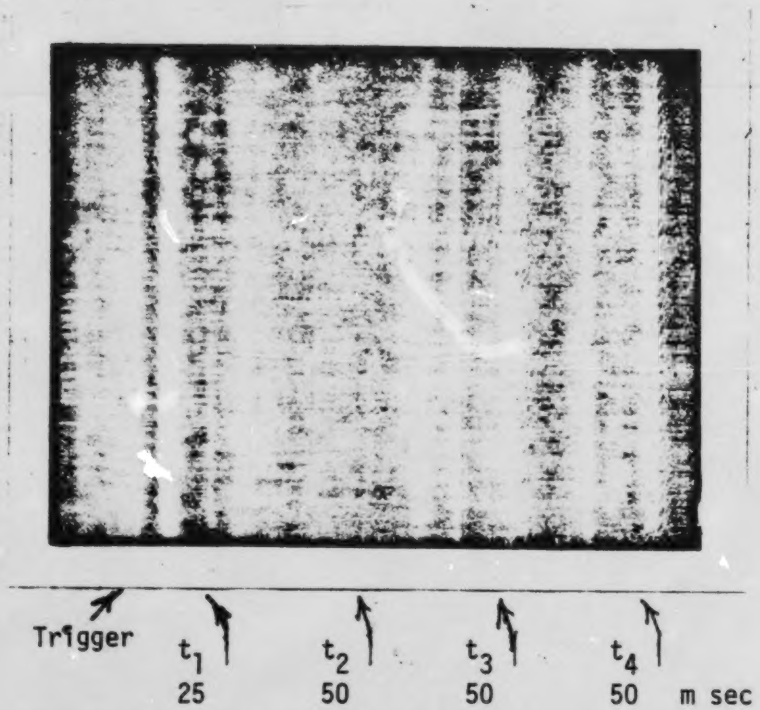
Alternatively one can buy the reverberation spring with transducers mounted on a simple frame with no amplifier from various electronic supply houses. Eg. Olson Electronics, Akron, Ohio, May 1974 less than $1.00.

Oscilloscope: Must have triggered calibrated sweep.

Piece of rubber foam.

Part A → Time





Measured spring length 8.8 cm. $t_2=t_3=t_4=2t_1$

$$v = \frac{8.8}{0.025} = \underline{3.52 \text{ m/sec.}}$$

$$\Delta f = \frac{v}{2L} = \frac{1}{2t_1} = \underline{\underline{20 \text{ Hz}}}$$

Parts B and C

Compressional Mode

Successive Resonances-stereo magic - (17 June '74)

	Freq.(Hz)	Height	Δf (Hz)		
1	18	small	18	- fundamental no nodes	All Broad
2	36	0.03		3rd harmonic 2 nodes	
			18		
3	53	0.08		2nd harmonic one node	
			17		
4	72	1.0			
			19		
5	91	1.2			
			19	resonances are broad.	
6	110	1.8			
			19	resonances are sharp.	
7	129	1.8			
			20		
8	129	2.8			
			20		
9	169	2.8			
			20		
10	189	3.4			
			19		
11	208	3.0			All Sharp
			20		
12	228	3.6			
			21		
13	249	3.4			
			19		
14	268	3.6			
			20		
15	288	3.4			
			20		
16	308	3.4			
			20		
17	328	3.8			
			2		
	330	1.8		= 20 Assigned to a non-compressional mode	
			18		
18	348	3.8			
			20		
19	368	2.0			
			3		
	371	1.0		= 20 Assigned to a non-compressional mode.	
			17		
20	388	4.0			
			20		
21	408	4.4			
			20		
22	428	4.6			
			19		
23	447	1.8		split 448 resonance not ampl. dependent.	
			2		
	449	1.8			

	freq.	Height	Δf
			19
24	468	3.0	
			19
25	487	3.4	
			19
26	508	4.4	
			20
27	528	3.2	
			18
28	546	3.2	
			19
29	565	3.2	
			20
30	585	2.6	
			20
31	605	3.2	

There is no particular reason to stop here - one can continue indefinitely.

THE AIR RESONANCE OF GUITARS AND VIOLINS

Object: To identify the air resonance of a guitar or violin and investigate its contribution to the sound quality of the instrument.

Apparatus: A guitar or violin, an opened beer or soda can (flip-top opening preferred), masking tape, a piano or a device to generate audible tones of measurable frequency (unnecessary if you have perfect pitch).

Procedure: Probably everyone has blown across an empty bottle and produced a tone of definite pitch (along with various amounts of hissing sounds). With a bit more difficulty, i.e., more hiss and less tone, blowing across a guitar sound hole or one of the f-holes in a member of the violin family will produce a tone referred to as the air tone, air resonance, or Helmholtz resonance of the instrument. The latter designation identifies the physical origin of the tone: the Helmholtz resonator is a somewhat bulky, rigid container enclosing an air cavity. The container usually has only one opening which is small compared to the dimensions of the container. A tone is produced when the mass of air surrounded by the small hole is forced to oscillate in and out over a small range. The volume of air trapped in the container acts as a restoring spring for the oscillating air mass. The pitch of the resonator is lowered if the area of the hole is decreased (most people guess the opposite will occur) or if the volume is increased. To observe these effects, blow across the beer can opening and note the change in pitch when: (1) the opening is partially closed with your finger, and (2) the can is emptied from being partially filled. The can is more suitable than a bottle because its characteristics are more like a stringed instrument and it is easier to observe the effects of reducing the opening size.

Now take the stringed instrument and determine the frequency of its air resonance by matching it with a piano note or other means. An alternate method of exciting the air resonance is to sing directly into the hole, sliding up and down the scale. Both f-holes of the violin are active although you need to blow across or sing into only one. By use of the second method, the instrument responds strongly at the air resonance with reinforcement of the sound accompanied by vibration of the wood body. Other resonances, usually higher in frequency, with strong wood vibration, can also be detected by the singing method. Like the air resonance, the frequency and strength of these "wood" resonances are important factors in determining the quality of a stringed instrument.

In most violins the air resonance is in the range C to D# near the open D string (260-300 Hz),A to B on the G string of a viola (220-250 Hz), and A to B on the G string of a cello (110-125 Hz). On a guitar its location is less easy to predict; some good guitars have the air resonance close to G on the lower E string (98 Hz). Once the frequency of the air resonance is identified you can observe the changes produced by partially closing with masking tape the sound hole of the guitar or one f-hole of the violin. Be careful to use tape that won't remove the finish on the wood. Cotton may be used in f-holes. With one f-hole of a violin completely closed, the air resonance moves down 3 or 4 semi-tones. Play the instruments under these conditions and determine which notes are most affected by the change. To discover why the instruments have sound holes at all, cover them completely and play the instruments. The instruments will be noticeably weaker on those notes in the vicinity of the air resonance which no longer exists.

By choosing the overall size of the instrument (volume) and the area of the sound hole(s), the instrument maker has determined the placement of the air resonance. If the sound hole of a guitar is too large, the instrument will lack bass response; if too small, it will be bass-heavy with poor treble response. One reason a cello is larger than a violin is to help achieve proper placement of the air resonance.

How does plucking or bowing the strings excite this air resonance? Recall that you could feel the vibration of the wood when locating the air resonance. Sound produced at the hole caused vibration of the wood. When the instrument is played, this process is reversed: the vibration of the wood causes sound to be generated at the hole. The wood vibrates because the bridge transmits the string vibrations directly to the wood.

One purpose of the soundpost in a violin is to properly place the frequency of the air resonance. Without the soundpost, the instrument sounds quite miserable, partly because the air resonance has been shifted down scale about 4 semi-tones. This lowering of the air resonance occurs because the instrument is hardly an ideal Helmholtz resonator with absolutely rigid walls. The presence of the soundpost stiffens the "container", which brings the instrument closer to the rigid state, presenting a stiffer enclosed air spring to the oscillating air mass in the sound holes. If a beer can were made of flexible rubber, it would have a lower air resonance than the identical size conventional metal can.

Just as in an organ pipe or wind instrument, standing waves can occur in the enclosed air of a stringed instrument. These "other" air resonances are less important than the Helmholtz resonance, but current research may reveal that knowledge of their relationships to the wood resonances help to identify a truly superior instrument.

REFERENCES

1. Carleen M. Hutchins, Alvin S. Hopping, and Frederick A. Saunders, "The Air Tone of the Violin," The Strad, September, 1959.

2. Carleen Maley Hutchins, "The Physics of Violins," Scientific American, November, 1962.

3. Michael Brooks and Donald A. Thompson, "Guitar Acoustics," Guitar Player, Vol. 8, No. 3, March, 1974.

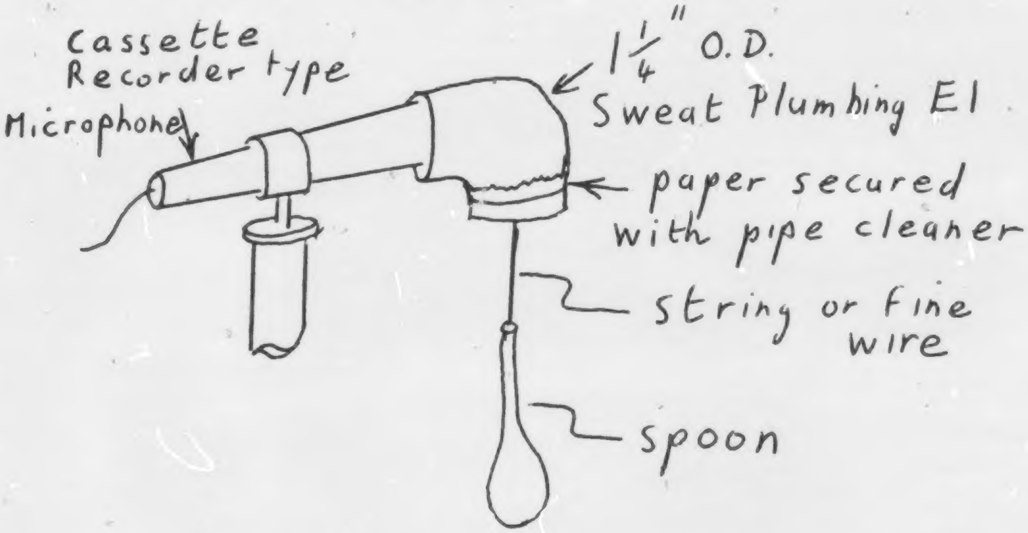
4. Carleen M. Hutchins, "A Note on the Function of the Soundpost," Catgut Acoustical Society Newsletter, No. 21, p. 27, May, 1974.

THE RESONANCES OF SPOONS

(also Forks, Knives, Small Wrenches, Hacksaw Blades, etc.)

I. Students may do their own experimenting by suspending a spoon, by the handle, in the middle of a string. Use thumbs to press the free ends of the string into both ears making sure that no other parts of hands or face touches the string. Now swing the spoon against the table edge or chair and enjoy a new sort of (stereophonic) high.

II. The following apparatus will recreate much of the above effect for large lecture demonstrations.


Cassette Recorder type Microphone
1 1/4" O.D. Sweat Plumbing El
paper secured with pipe cleaner
string or fine wire
spoon

Wrap the inside of one end of a 1 1/4" OD, 90^0 sweat copper plumbing el with soft tape so that the Dynamic Microphone fits snugly inside with the front end of the microphone almost up to the bend. The other end of the el will be covered with a diaphram made from a piece of ordinary paper secured with a pipe cleaner. The spoon is suspended from a length of string not more than 3" long which is inserted through a small hole punched in the diaphram and restrained by a knot. The microphone goes to preamp, power amp and loudspeakers. A quad decoder at "line" level may give an interesting stereo effect.

The most critical element of the system is the string connecting spoon to diaphram. Using soft twine is mechanically simple but severely attenuates upper partials. Note: Since the harmonics are presumably exponentially attenuated the ultimate loudness of upper partials should decrease approximately linearly with string length. Wire which swings from the diaphram without bending when the spoon swings (eg. #22 copper wire) can be used but it must be glued to the inside of the paper diaphram to avoid an intolerable scratching noise. The other end of the wire must be wrapped tightly and several times around the spoon handle to avoid scratching noise at that end. The advantage of wire is that more of the upper partials are transmitted. Apparently the finer the wire the better this works.

Caution: Because of the large amplifier gains ($\simeq$ 2000 from preamp alone) the system should not be jarred. Best to do the demonstration in the middle of the lecture when people are not coming in or going out.

Note on diaphram: A stiff cardboard diaphram does not respond well to high harmonics and produces a smaller signal. Paper works better. Put the hole in the paper diaphram OFF center.

Results: Do not bang the spoon; tap it with various small objects. Try a metal screwdriver tip and the corner of a pine block. Of the eating utensils the spoon gives the most interesting sounds. Strike at different points on the perifery of the ladle. Strike the edge of the handle and the bend at the throat. The principal resonance should be diminished relative to upper partials when the spoon is struck at the throat. The principal resonance is strongest when the spoon is struck at either end. Interesting result on a three-tined fork: On the Dansk V large fork I hear beats between normal modes which apparently correspond to vibration of the two outer tines. On a four-tined fork there is quite a jangle in the region of the tine resonances.

The Mixing of Colors

Notes to instructor:

It is highly unlikely that you would have more than 3 slide projectors at your disposal, so it is assumed that you will be setting up the situations in this experiment while the students, each with a diffraction grating (or a hand held spectrometer) will observe and record their observations.

It is suggested that you do the experiment at least once ahead of time so that you may see how to set up the projectors for optimum illumination and so that you may see if your prepared reflecting surfaces actually give the desired results.

Note that the heat filters and bulbs vary from projector to projector. In several sections of the experiment, it may be possible to switch which filter is used in which projector to give the best result. Also make sure your lamps are set to the same brightness.

Edmund card mounted diffraction gratings (catalog # P-30,282 for 40) are so inexpensive that you can give each student one. These gratings seem to give a brighter spectral image than the hand-held spectroscopes. If you wish to make the experiment more quantitative, you may just prop a meter stick against the screen where the spectra are to be viewed. Science Kit makes a hand-held spectroscope with a numerical scale inside it (catalog # 16525) but the spectra are fainter.

Colored filters may be purchased from Welch already in cardboard slide mounts. You may choose to order the large sheets from Edmunds, however, and make your own slides. You get more that way and you can use strips of the same color in the next title experiment on Spectra of Colored Lights.

It is easier to view the spectrum if you are looking at just a slit of light. A piece of cardboard with a slit about 3 mm wide can be inserted into the slide projector with each color filter. The cardboard should be sturdy and should be the same size as a standard slide.

Sections A and B are self-explanatory. If your students have never used a diffraction grating, you may wish to have them sketch the spectrum from a white light source before beginning section A.

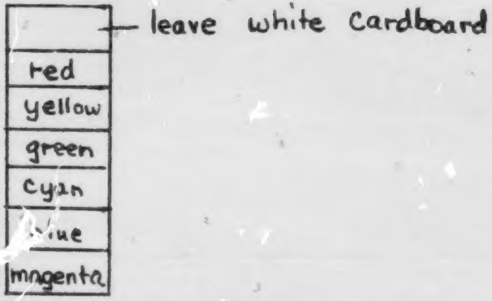
Section B has an optional section. This was considered less important and was made optional out of time considerations.

Section C needs several cards to be prepared by you prior to the day of the experiment. The size of the cards and of the painted sections depends largely on the intensity of your projector. The main suggestion is to try it and see if it works. If not, make any necessary changes (cut wider slits, move projector closer to screen, etc.)

You will need three or four painted cards. Our results were best with the main suggestions given. Some alternate suggestions will be made.

We used heavy cardboard about 3" x 12" and painted bands of color one below another about 3/4" to 1" high with acrylic paint (alternate suggestion - tempora paint.) If you use acrylic, you will get a smoother finish if you dip your brush in water slightly before applying the paint. You will need to purchase at least 8 tubes of acrylic paint - red, yellow, green, blue, cyan, (blue-green), magenta (or red violet), two other greens.

Card 1 will look like this:



Card 2 will be the same only the color bands will be various shades of green (all acrylic). Use some shades straight from the tube then mix one green with yellow and cyan and another with yellow & blue. (alternate suggestion - use various shades of yellow including orange, chartruse and gold.)

Card 3 will have color bands of the same color but of different media. We suggest bright yellow as it is fairly easy to match. Some media to consider are: artists quality - watercolor, acrylic, tempora, oil; cheaper quality - enamel, spray enamel, magic marker, paper, etc. We got good results for easy discrimination using spray enamel, magic marker and acrylic.

You may wish to have a fourth card with just a red and magenta, or you may use Card 1 for the last section of the experiment. The magenta should be tested for the last section; it should look like a true red when viewed through a yellow filter and like a true blue when viewed through a cyan filter. We found artists quality of tempora paint to work well here - color Red Violet.

Section C also has two optional sections. These were again made optional partly because of time considerations and partly because discrimination of differences in the spectra is exceedingly difficult. It is to be noted, however, that these topics are particularly relevant to the art student, so that if you think discrimination can be made and if you have time, it is suggested that you include these optional sections.

If you have time, you may wish to let your students mix their own pigments to look at through a grating.

The introductory question regarding how spectral analysis is used to detect forgeries can be discussed with the students thusly: certain pigments were not used to make cobalt blue until the 16th century. If a spectrum analysis of a painting supposedly done by a master in the 14th century shows that Cobalt blue was used, the painting is obviously a forgery.

Suggested references (the first listing is the best for this experiment).

1. Colour - Its Principles and their Applications by Frederick W. Clulow; Publ. - Fountain Press, 46/47 Chancery Lane, London; 1972.
2. Light and Color by Rainwater; Publ. - Golden Press, Western Publ. Co.; paperback.
3. Eye and Brain: The Psychology of Seeing by Richard Gregory; Publ. - McGraw Hill; paperback.
4. Seeing and the Eye by G. Hugh Begbie; Publ. - The Natural History Press, Garden City, N. Y. - paperback.
5. An Introduction to Color by Ralph M. Evans; Publ. - John Wiley & Sons.
6. Color Vision; National Academy of Sciences, Washington, D. C.

OBSERVATIONS ON THE SPECTRA OF COLORED LIGHTS AND OBJECTS

OBJECT: To observe the light from different light sources, light which has been passed through colored filters, and light which has been reflected from colored surfaces, to see that light of a given "color" may arise in a variety of different ways.

MATERIALS NEEDED:

Long filament clear tungsten light bulb, (such as is used in many display cases).

Gaseous discharge tube and a means of exciting it. An uncoated flourescent tube (from Edmund) can be used here; it is a mercury source.

Diffraction gratings, one for each student; cheap Edmund gratings will do.

(These gratings may be more useful if part of a cheap spectrometer, such as obtainable from "Scientific Kits." They may also be used with homemade spectrometers constructed from rectangular breakfast cereal boxes, like Wheaties, etc.)

Various colored filters. Available from: Rosco Laboratories, Inc.
Port Chester, N. Y. 10573

Theatrical filters are cheaper than photographic filters, and just as good for the purposes of this experiment.

Variously colored construction paper, especially of the lighter hues. Samples of flat painted surfaces, fabrics, etc. are also useful.

Slide projector, or other means of producing strong uniform illumination on a surface. Perhaps the image of a frosted tungsten filament lamp can be used here.

PROCEDURE:

Holding the diffraction grating directly in front of one eye, look through it at the source of white light (a tungsten filament lamp bulb). Notice the bands of color to the right and left of the source. The hues seen in the innermost bands (the "first order" spectrum; we shall ignore the second and higher orders) includes violet to red (VIBGYOR) proceeding outward from the central image. The presence of these bands of color indicates that the so-called white light emitted by the bulb is in fact a mixture of light having all the various hues observed in the band. Carefully note that the band of colors contains no gaps; every hue is represented. Such a spectrum is called a continuous spectrum. Make a sketch of the continuous spectrum observed, noting the approximate wavelength (in nanometers or in Angstrom units) for each major hue.

Now look through the diffraction grating at a gaseous discharge tube (mercury, helium or other), noticing that the spectrum now consists of a discrete set of line spectra, and indicates that the light from the discharge tube, in contradistinction to that from the tungsten lamp, consists of only a few discrete hues. The marked contrast between the two kinds of spectrum, continuous and discrete, is due to the differences in details of the physical processes involved in light emission from the two sources. Note further that the observed color of the source as seen without the diffraction grating has no obvious connection with the spectrum observed. For example, though the mercury light appears bluish to the unaided eye, the diffraction grating shows it actually to contain red, yellow, green and violet lines in addition to lines of bluish hue.

Next let us observe the spectrum of light transmitted through gelatin filters of various colors. To do this we send white light from a tungsten filament light bulb through the filter, and inspect the spectrum of the light which survives the journey. We make use of strips of red, blue, and green filter material about one inch wide wrapped around a long filament lamp. By using the diffraction grating in the manner previously described, all the spectra from several filters can be observed simultaneously, and compared with the spectrum of white light. It will be noticed that the light which comes through a filter of a given "color" is not all of the same hue, and in fact it may contain little or no light of the hue ascribed to the filter name. That is to say, the light transmitted by a filter is not monochromatic, but a mixture of various hues which together produce the same sensation as would be produced by a monochromatic light of a certain hue. The name given to this fictitious monochromatic hue is the same as the name which is ascribed to the filter (as red, blue, green, etc.)

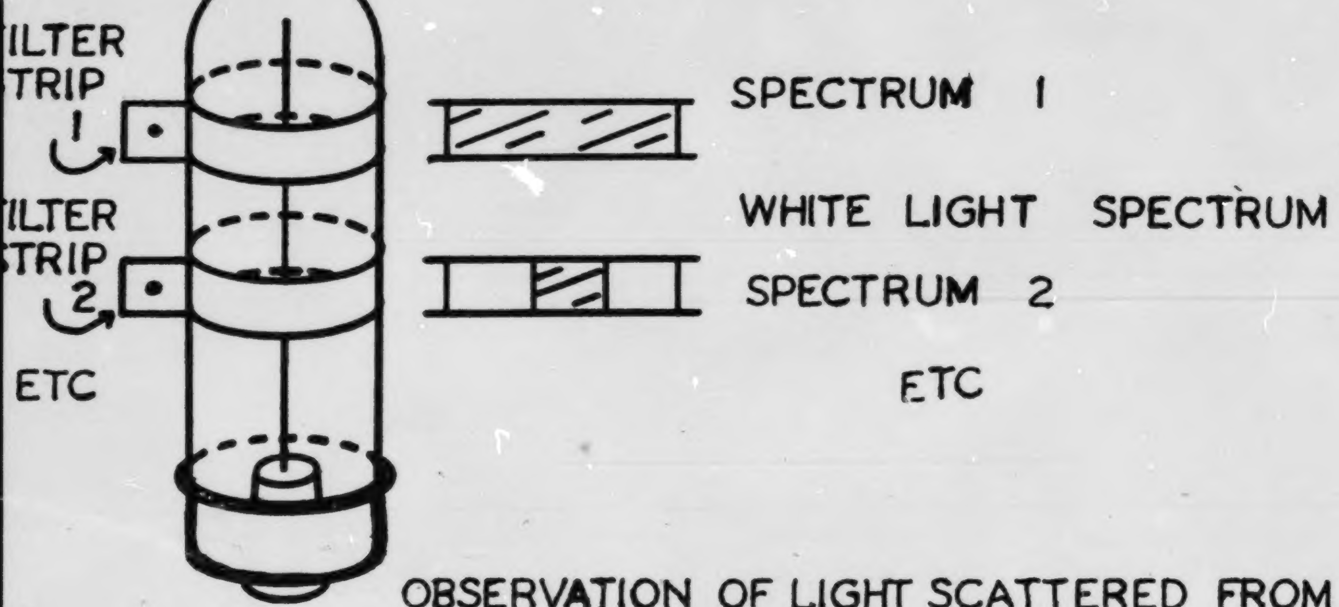
Next, by following the same procedure as above, we observe the light transmitted from a tungsten filament source through several filters of various shades of red (for example, dark red to orange). A pencil sketch should be made which indicates the region of transmission of each filter, and it should be carefully noted that the differences in color of the filters may be due to fairly subtle differences in the spectrum of the transmitted light.

Finally we want to observe the spectrum of white light scattered or diffusely reflected from a colored surface. The experimental problems likely to be encountered here are too low intensity of the scattered light and specular reflection from the surface. The first will make the spectrum difficult to observe, and the second will give rise to an unwanted continuous background. It is convenient for this experiment to use a series of strips of construction paper of various hue, arranged in a vertical array, and illuminated by a strong vertical band of white light. The spectrum of scattered light can then be observed in the way previously described for the filters.

A convenient source of light is a slide projector, with a slotted plate in the slide holder. The vertical stripe of light should be narrower than the samples illuminated. Another possible light source is the vertical filament lamp of the previous sections focussed on the samples with a fresnel lens, or better with a cylindrical lens if one is available. Intensity of illumination is likely to be a problem in any case, and it is for this reason probably necessary to darken the room, or to observe the spectrum through a "cereal box spectrometer". Various other samples can be used among which should be included certain flat paints, some brightly colored fabrics ("day-glow"!) and paints, and also some natural materials like leaves and flowers. A sketch should be made of the spectra obtained, and any peculiarities noted.

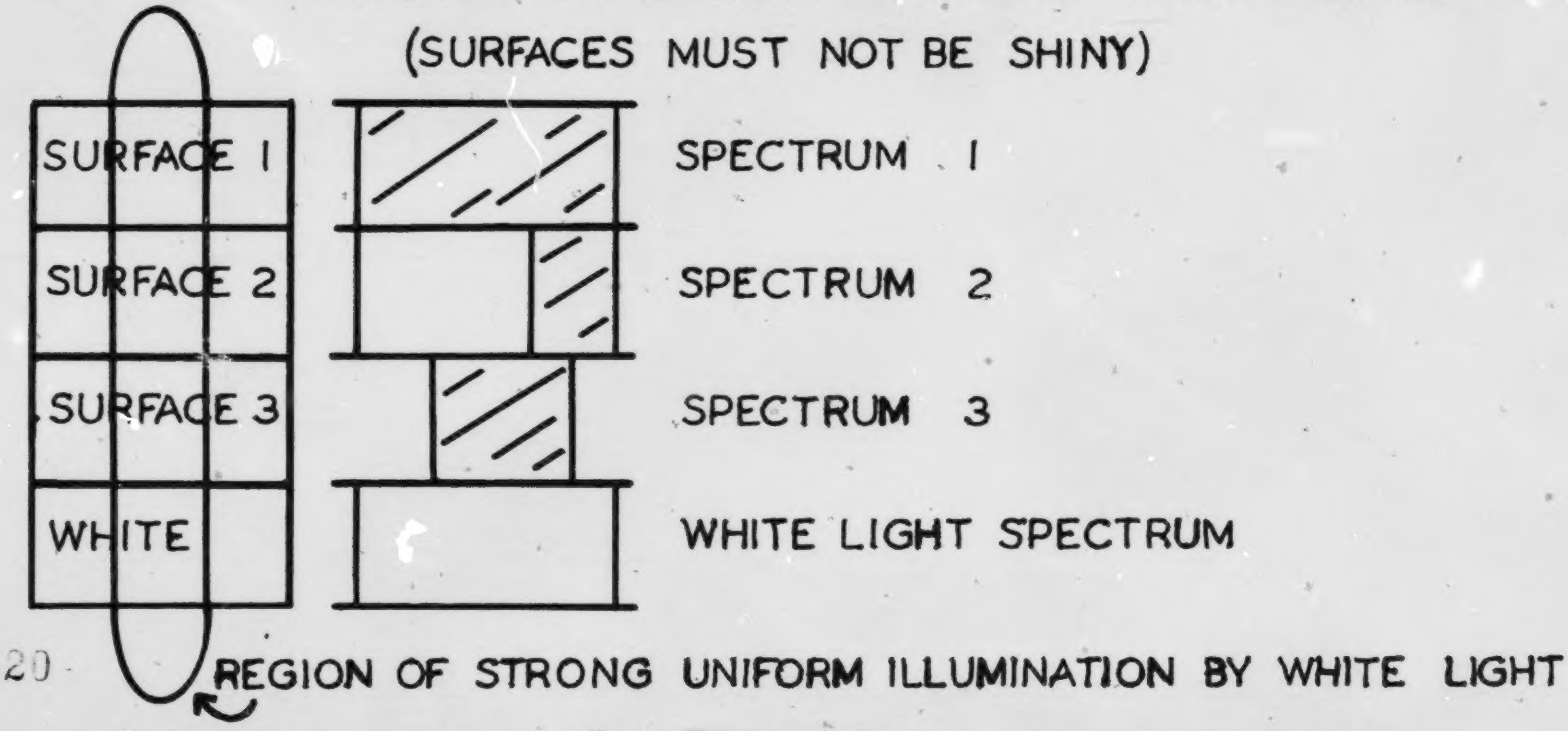
TAKE-HOME EXPERIMENT: Obtain a diffraction grating and with it observe the spectra of various lights encountered in everyday life. Red and green traffic signals, flourescent lights, neon signs, sunlight, starlight, etc., are good subjects. Cereal box spectrometer is a convenient aid for making some of these observations.

OBSERVATION OF LIGHT TRANSMITTED THROUGH FILTERS


ILTER
TRIP
1
ILTER
TRIP
2
ETC
SPECTRUM 1
WHITE LIGHT SPECTRUM
SPECTRUM 2
ETC

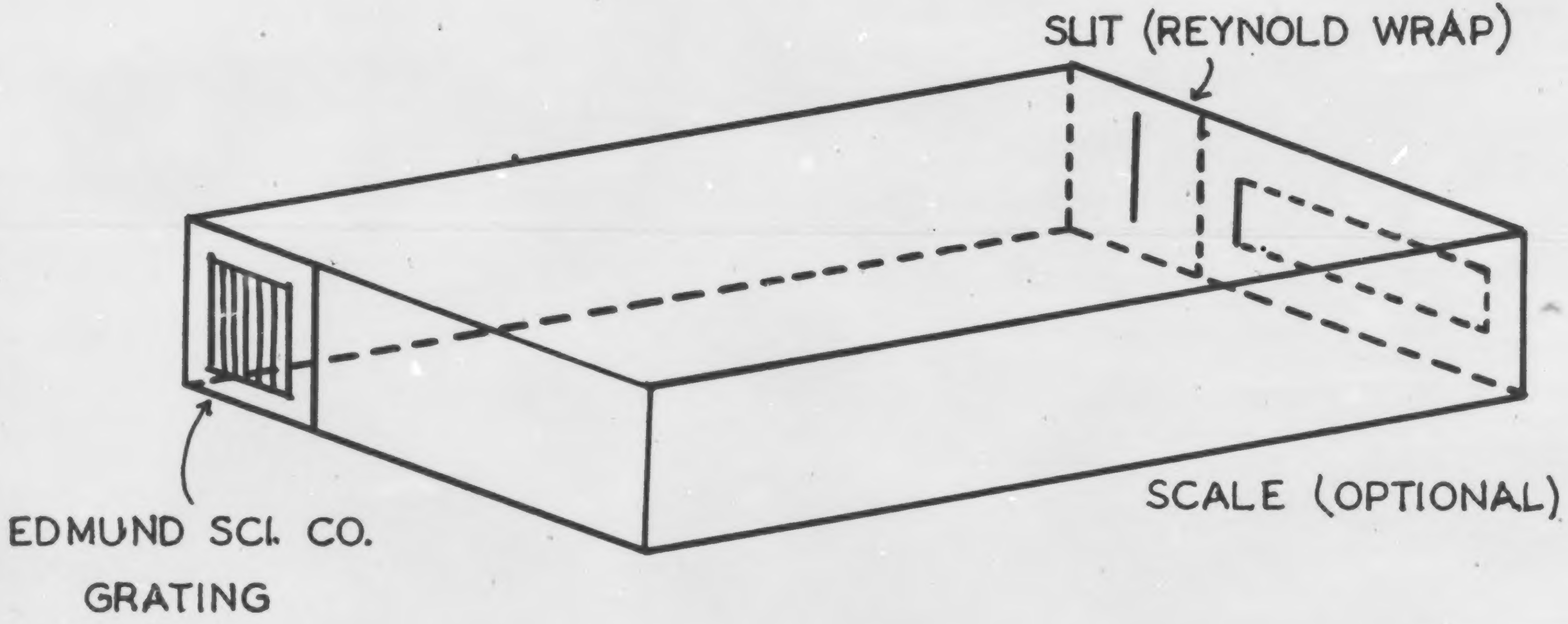
OBSERVATION OF LIGHT SCATTERED FROM COLORED SURFACES

(SURFACES MUST NOT BE SHINY)


SURFACE 1
SURFACE 2
SURFACE 3
WHITE
SPECTRUM 1
SPECTRUM 2
SPECTRUM 3
WHITE LIGHT SPECTRUM
REGION OF STRONG UNIFORM ILLUMINATION BY WHITE LIGHT

32.5

CEREAL BOX SPECTROMETER


SLIT (REYNOLD WRAP)
SCALE (OPTIONAL)
EDMUND SCI. CO.
GRATING

32.6

The Mixing of Colors

Apparatus: 3 slide projectors (one set per class), colored filters,
colored cards, diffraction grating, good eyes.

Introduction: Did you know that a color TV transmits only red, green and blue colors - yet you see every color of the rainbow when you watch it? Do you know why a painting looks different in artificial light than it does in sunlight? And do you understand what it means to use "spectral analysis" to study whether an "old" painting is a modern forgery or not? Let's look at some of the processes which will relate to the above questions.

Two of the questions above relate to the mixing of colors. We will use "spectral analysis (in a crude, but interesting, way) to study the mixing of colors. There are at least four ways to mix colors. The most obvious way to the painter is to mix two pigments together to get a shade he may not be able to buy. One of the obvious ways to a physicist is to mix 2 beams of light having different wavelengths to get a resultant color which is different from the two he mixed. It is also possible to place two different filters together in the path of a beam of white light and get a color projected on a screen which is different from either color filter. And finally we can view colored objects by using non-white light.

In this experiment you will be able to mix colors in each of these four ways. Since each process is slightly different, the theory for each process will be presented in turn. There is no numerical data to read, but you must report your observations clearly for each section. A suggestion on how to do this report is given in the first section of the procedure. It is also suggested that you write four summaries - one for each way of mixing colors. This summary should try to tie together the various observations you have made in that part of the experiment.

All of your observations will be made by looking through a diffraction grating at a slit of light projected on a screen. You will be expected to observe and record the spectrum that results each time, as well as to answer any questions that are presented.

A. Mixing of "pure light"

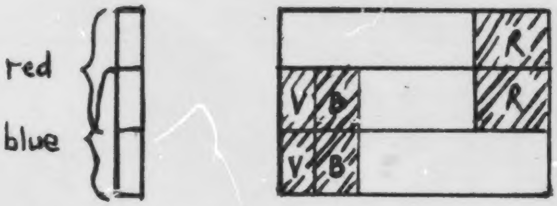
Theory: It has been found that all possible colors of the white light spectrum can be matched by overlapping only 3 basic wavelengths (or bands of wavelengths) - those being blue, green and red. These are, in fact, the three colors used in your color TV to duplicate all other colors. Although the processes of color vision are not yet clearly understood, it does seem that your eye responds to the mixtures of red, green and blue to produce sensations which duplicate all colors you can normally see. And, in fact, when these three colors are projected together, one sees white light. In this case, the white light is not composed of all wavelengths of light, yet it gives the same sensation to the eye as if all wavelengths were present.

When "pure light" is overlapped in the process described above and performed below, this is called the addition of colors and the three colors red, green and blue are called the primary colors for pure light. (This may be seen to be different from the primary colors for pigment or paint.)

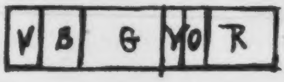
Procedure:

1. Set up three slide projectors aimed so as to overlap on a screen. Place a red filter viewed through a narrow slit as a slide in one, a green filter and narrow slit in another and a blue filter and narrow slit in the last.

2. Add each two of the primary colors to get the secondary colors. As mentioned above, you will be viewing each section through a diffraction grating and recording the spectrum that you observe. It is good to align the two beams so that you can see each color separately as well as see the overlap region. It is suggested that you record the right hand spectra in the following manner:



This is based on a spectrum of white light which would look approximately like



3. Record each of the spectra in a manner similar to that above. Make note of any colors that are less intense than the others. The mixtures to view are:

a) red and blue produce magenta

b) blue and green produce cyan

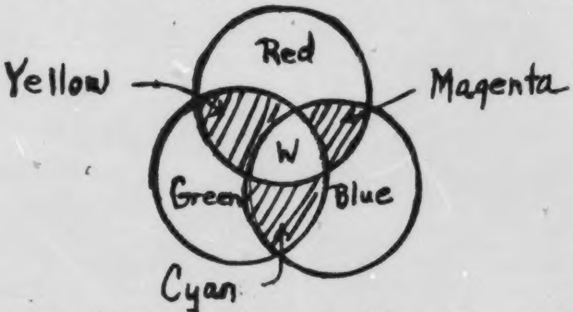
c) red and green produce yellow

The three colors magenta, cyan and yellow are called the secondary colors for light.

NOTE: Keep the record of these spectra handy. You may need to refer to them to explain some of the effects seen later in the experiment.

4. Add all three primary colors to get white light. Compare this white to a white light source from a long filament incandescent tube. What do you conclude from this comparison?

Theory: Often a color wheel is used to represent what you have just seen. The three primary colors are shown overlapping to produce the three secondary colors. Where the three colors all overlap, white light is produced.



Any two colors opposite each other on this color wheel are called complementary colors. When two beams of light from complementary colors overlap, white or rather whitish light is produced. This is easy to explain: for example, when yellow light and blue light are allowed to overlap, the yellow is actually a band of red, yellow and green wavelengths as you saw in part (3) above. If you add blue wavelengths to that, you have all wavelengths, hence you perceive white light. However, the relative proportions may differ, just as white sunlight, in candescent light and fluorescent light differ.

Procedure:

5. Add each two of the complementary colors. Record the spectra observed in each case and record your subjective impressions. (NOTE: You may not get a perfect white because of the transmission band width of each color filter but the result should be convincingly lighter than the two overlapping colors).

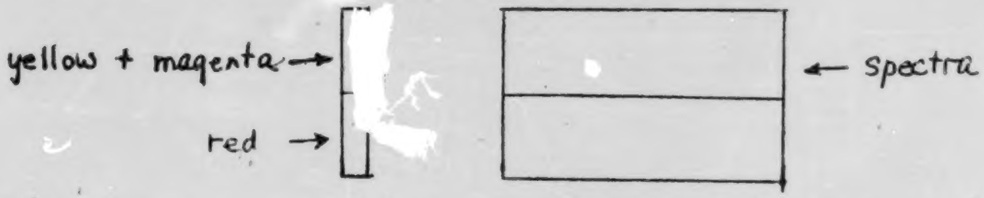
6. Write a summary for this section as to how "pure light" mixes.

B. Mixing of filters

Theory: When a filter is placed in a beam of white light, it absorbs some wavelengths and transmits others. Thus a red filter transmits red wavelengths and absorbs other wavelengths from the white light spectrum. This process of absorbing some wavelengths is called a subtractive process. When a filter of a secondary color is used, it absorbs in a small region of the spectrum and transmits over a range corresponding to at least two primary colors. For example, a yellow filter absorbs blue wavelengths (subtracts blue wavelengths) from white light and transmits red, yellow and green. When two filters are placed together, each one subtracts a segment of the white light spectrum and only that portion transmitted by both will be projected onto a screen.

Procedure:

1. Place two filters together so that light must pass through two secondary colors. Use another projector to compare the mixture of secondary colors to those of the primaries. For example:



Repeat for yellow and cyan compared to green. Repeat for magenta and cyan compared to blue. Explain why you are getting the colors that are produced on the screen. Do this by explaining which wavelengths are absorbed by each filter and, therefore, which wavelengths are not absorbed but are transmitted to the screen. (NOTE: It would be possible to exactly match colors if you had the correct filters.)

2. Describe what you see when light is projected through each pair of primary filters. Explain what you see in terms of absorbed and transmitted light for each pair of filters.

3. Optional: You may wish to look at combinations of secondaries with appropriate primaries: e.g., a green filter alone gives the same color on the screen as a green and cyan placed together. Can you explain why this is so? Try it with other filters for a similar effect. Record your results and explain what is happening.

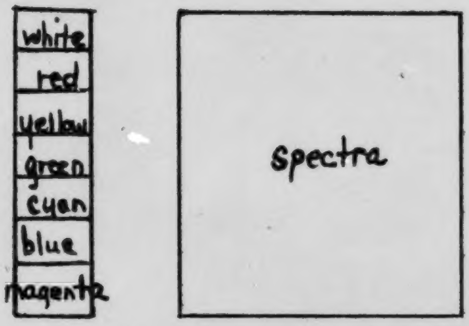
C. Mixing of pigment -

Theory: The way in which we view an opaque object is similar to the way we see a filter: we see the wavelengths of light which are not absorbed. The wavelengths not absorbed are, in this case, reflected. Again this is a subtractive process since we see only what is not subtracted (or absorbed) by the opaque object. Consider the pigment used in bright yellow paint: the paint appears yellow because it is reflecting red, yellow and green wavelengths of light to our eyes. Therefore it is absorbing the blue wavelengths. If we mix this yellow paint with a cyan paint (which reflects blue and green but absorbs red and yellow) all wavelengths are absorbed by one pigment or the other except for green so green is the color we see reflected. If, instead of cyan (which is greenish-blue) we use other shades of blue to mix with yellow, we vary the shade of green we obtain. (You may wish to try this). Similar results are obtained by mixing other pigments. To predict what the resulting mixture will be, keep in mind which wavelengths are absorbed by each of the pigments you are using to mix.

Procedure:

1. Set up card 1 (to be prepared by instructor) and illuminate it with a slit of white light so that you see light reflected off of

each of the colors on the card and also you have some white across the top as reference. Record the spectra you see. Is this what you would expect? Compare these spectra to those of the light transmitted through the colored filters. What do you conclude?



2. Set up card 2 and repeat your observations and method of recording. These colors may be harder to discriminate as they are shades of one or two colors. There are differences, however, and that is why the colors look slightly different - they are reflecting slightly different wavelengths of light. Compare the colors you see with the naked eye to the spectra you obtain. Are the spectra what you would expect?

3. Set up card 3 and repeat the above procedure. These colors are all the same (or as close as was possible), but they are made from different media (i.e., watercolor, oil, enamel, etc.) Your instructor will tell you what each is. Can you discriminate any differences? Show the spectra and discuss what you see.

4. Optional: You may wish to repeat the above procedure with card 1 with different light sources (i.e., fluorescent, incandescent, ultraviolet, sunlight, candlelight, etc.) Show the spectra and discuss what you see.

5. Optional: You may wish to repeat the above with light reflected from colored paper or other colored objects especially to see what effect texture may have. Show the spectra and discuss what you see.

D. Viewing colored pigment through colored light

Theory: When white light is projected through a filter, the filter absorbs some of the wavelengths. If the transmitted light is allowed to fall on a different color pigment, still more of the wavelengths will be absorbed and only a few will be reflected back to the eye. This is similar to what happens when an artist puts a glaze over a painting. It is also similar to what happens if you view a picture in non-white light.

You should be able to predict what will happen with different colors. Let's do one example: suppose we have red pigment on a card. It is red because it reflects only the red wavelengths; it absorbs all others. What would happen if it was now illuminated by cyan light (i.e., white light which has been passed through a cyan filter?) Since cyan light has only wavelengths in the blue and green region of the spectrum and since these are absorbed by red pigment, there should be no reflected light and the pigment should appear black.

Procedure:

1. Try the above example and see what happens. Look just with the naked eye, then view through a diffraction grating. Record any spectrum you see.
2. If you project yellow light onto a magenta pigment, you should see a red color reflecting off. Explain why the color you see should be red, then set it up and see. Record the spectrum and discuss your observations.
3. Use the same magenta pigment as above and predict what will happen if you project cyan light onto it. Explain what you expect to happen and why. Only after you have predicted what will happen,

set it up and see. Again record the spectrum and discuss your observations.

Appendix to the Color Mixing Experiment: The Intersection-Union Theory of Color Mixing

Students familiar with the "new math" concept of set theory (union and intersection) should be able to understand color mixing in the following way. We start with the three basic optical colors: blue, red, and green. Denoting them by the letters B, R, and G respectively, we will symbolize what is commonly called "color addition" by the union sign $\cup$. Therefore $B \cup R$ is called magenta, $B \cup G$ is called cyan and $R \cup G$ is called yellow. $B \cup R \cup G$ is our universal set U which is called white. As we found by experimentation, color union is effected by superimposing directly these spectra responses (by direct light). The "Pointillism" phenomenon (and the color T.V. tube) creates the same effect by juxtaposing these colors in small dots. "Color subtraction" which is observed by multiple filters, pigment mixture, etc. corresponds to set intersection. Color wheel phenomena are then explained in terms of the laws of set theory using the rules that $B \cap G = B \cap R = R \cap G = \phi$ (the empty set) which we call black. This means that when multiple filters of red and blue are combined, no light is visible since their spectral distributions are disjoint. For example, let us see what happens when yellow and cyan pigments are mixed. We obtain the formula: $(R \cup G) \cap (B \cup G) = (R \cap B) \cup G$ (by a distributive law of set theory) $= \phi \cup G = G$ so that green results. Note that union of white always gives white (since it is our universe) and intersection of white with another color gives that same color back.

Color complements are then their set complements (i.e. the colors whose union is white and intersection is black). The complement of red is cyan $(B \cup G)$

since

$$R \cup (B \cup G) = R \cup B \cup G = U \quad \text{(white)}$$

while

$$R \cap (B \cup G) = (R \cap B) \cup (R \cap G) = \phi \cup \phi = \phi \text{ (black).}$$

All other phenomona we observed have similiar set theoretic interpretations.

The Camera & The Projector

Purpose: to investigate some of the image forming properties of a thin converging lens.

Apparatus: a simple camera, optical bench with light source, holders, etc., two lenses: one long and one short focal length, 35 mm slides as objects.

Introductory exercise: Set up an adjustable focus camera having a simple lens, a shutter that can be held open and a translucent screen at the rear where the film would normally go. Look at some objects out the window and see how the lens works to make an image on the translucent screen. Note that to view objects at different distances you must adjust the length of the camera box to get a sharp focus on the screen (or film). If you have a 35 mm camera or a good Polaroid camera, adjust the focus on it and again notice how the focus adjustment causes the length of the box (the distance between the lens system and the film) to be changed.

Take a 35 mm slide and project its image onto a large screen using a projector. Notice again how you focus the image - you adjust the distance between the lens system and the slide.

Both of these optical devices rely on lenses in order to form images. They are, in fact, "opposite" instruments: one takes an object from far away and focuses a small image onto film; the other takes an object nearby (the slide) and focuses a large image onto a screen fairly far away.

Let us disgress briefly and look at the theory of how lenses "work", then we will return to the camera and projector so that you will understand how the lens is used in each.

Theory: A converging lens is one which is thicker at the center than at the edge and converges incident parallel rays to a real focus on the opposite side of the lens from the object. A diverging lens is thinner at the center than at the edge and diverges the light from a virtual focus on the same side of the lens as the object.

If parallel light rays (such as those from the sun or those which have reflected off of a far away object) are allowed to pass through a converging lens, the rays are refracted and will meet at a point. This point is called the focal point, F. The distance from the center of the lens to the focal point is called the focal length, f, or focal distance

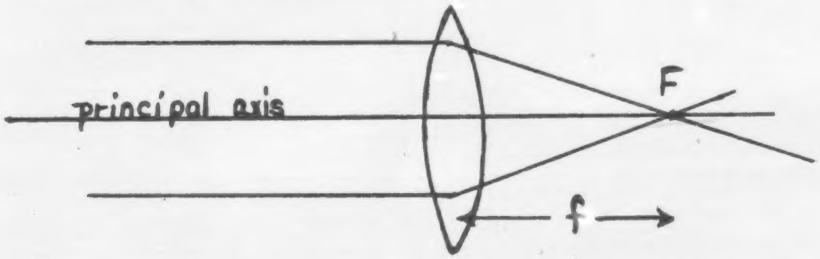


Fig. 1

The principal axis of a lens is a line drawn through the center of the lens perpendicular to the face of the lens.

The light rays are reversible. If a point source of light is placed at the focal point of a lens, the rays refract in the lens and travel as parallel rays never forming a focus. They are said to form a focus at infinity.

We view things near to us by the light which reflects off of objects and this light is rarely parallel. It is still possible to focus these light rays with a lens but the focus is not usually at the focal length. We call the distance of the object to the lens the object distance, OD, and the distance from the lens to the focused image the image distance, ID.

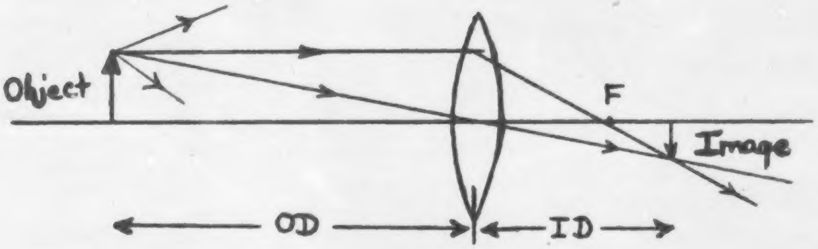


Fig. 2

To find where the image is located, we can draw a ray diagram. The object has many light rays reflecting off of it, some of which pass through the lens to create the image. We can not predict the path of all of these rays, but two of them we do know about. There will be one ray parallel to the principal axis and it will pass through the focal point just as the parallel rays from the sun passed through the focal point. Then there is one ray that goes through the center of the lens. It is essentially undeviated. Where these two rays intersect is also where all the other deflected rays will intersect and a focused image will be formed.

Another way to find the image position is to make use of the equation:

$$\frac{1}{f} = \frac{1}{OD} + \frac{1}{ID}$$

where f is the focal length, OD is the object distance and ID is the image distance, all of which were defined above. In this experiment all of these values will be positive.

Procedure:

1. Take a lens of short focal length (5cm-10cm) for a study of simple lens phenomena. If you don't know the focal length of a lens, a fairly good way to find it is to let light which is reflecting from a far away object (a tree or telephone pole outside) to pass through the lens and come to a focus on a screen. The distance from the lens to the screen can be measured and is

the focal length, f. (This corresponds to Fig. 1) Try this by holding the lens in one hand and the screen in the other, but for accurate measurements support them on an optical bench. Record the value of f.

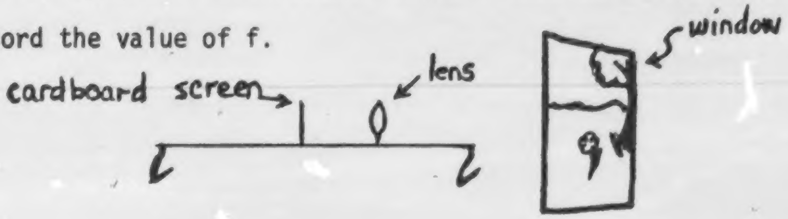


Fig. 3

2. Make some observations regarding object position, image position and image size by using the lens you used above and setting up the optical bench like this:

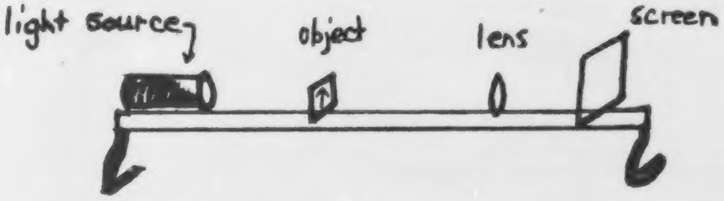


Fig. 4

a) Place the illuminated object at a distance greater than 2 times the focal length from the lens (OD > 2f). With the cardboard screen on the other side of the lens, find the image. Is ID

1) less than f (ID < f)

2) between f and 2f (f < ID < 2f)

3) equal to 2f (ID = 2f)

4) greater than 2f (ID >2f)
(Place an x opposite the correct value)

Also notice the size of the image relative to the object. Is it
1) smaller than the object

2) same size as the object

3) larger than the object

b) Repeat the above observations for these values of OD and report your observations.

OD	ID	size of image
1. OD > 2f		
2. OD = 2f		
3. OD > f but OD < 2f (f < OD < 2f)		

c) Can you see a pattern to what is happening? If not, run through it again until you do. Describe this pattern.

3. Camera - The simplest of small cameras use a single convergent lens similar to the one you have been using. A more complex camera uses several adjacent lenses to reduce certain undesirable effects found with a simple lens. For our purposes, we can make a replica of a camera with just the single convergent lens you have been using.

A camera takes light rays from an object that is fairly far away and focuses this light through a lens system onto film which is close to the lens. A good camera has some slight adjustment to where the film will be placed relative to the lens.

a) Illuminate an object on one end of your optical bench (say 60-70 cm). Calculate what the image distance, ID, should be by using the lens equation and your values of OD and f. Then find the image on the screen and measure ID. Calculate the %-difference between your calculated value and your measured value. Do this for at least 3 different object distances extending to more than one meter if possible.

The system you have just set up is a simple single lens camera without the covering black box. With a box of some sort to keep out stray light, a shutter to let light through the lens for only part of a second and film at the same position as the screen, you would be able to take pictures.

Some of you may have an Instamatic or a similar camera which does not have an adjustable focus on it (i.e., no way to vary the image distance.) Let's see how this type of camera works.

b) Repeat the procedure in (a) above at least 3 times only now use objects that are at least 3 meters from your lens. You may need to illuminate the object and turn out the room lights to see a good image. Use a metric tape measure to measure the object distance. Calculate what ID should be by using the lens equation and your values of OD and f. Then find the image on the screen and measure ID. Calculate the %-difference between your calculated value and your measured value.

Compare the three ID's to each other and to the focal length of your lens. What similarities do you see? Do you see how a box camera like the Instamatic works? Explain.

4. Projector - In its simplest form, a projector is another single lens optical instrument. The illuminated object (say, a 35 mm slide) is placed close to the focal point of the lens (upside down so that the image is seen right-side-up.)

a) Place your optical bench several meters from a projection screen. Record the image distance ID that you will use measured from the screen to your lens on the optical bench. Calculate the object distance OD necessary to have this ID, then set up the illuminated object and adjust its position until you have a sharp image on the screen. Record the measured OD and calculate the %-difference between your calculated value and your measured value. Do this for at least 3 different image distances.

If a projector is normally used a long distance from a screen, a lens with a long focal length must be used to prevent the image from being too large and dim.

Most projectors use several adjacent lenses to reduce certain problems found with only a single lens, but the effect is the same.

Also most projectors use a lens system called a condenser to focus the light source so that the light rays are nearly parallel (actually focussing at the center of the projection lens) when they pass through the slide. (Remember that this is the inverse of Fig. 1 - the light source is put at the focal point of the lens and the rays refract in the lens so that they leave parallel to the principal axis.) Most projectors have a good point source of light. Your source is an extended source so it is hard to put it right at the focal point, but approximately at the focal point will allow an optimum of light intensity on the slide.

b) Use your short focal length lens as a condenser and find a lens with a longer focal length to use as a projecting lens. Place the light source at the focal point of the condenser lens. Place the slide to be illuminated (upside-down) just in front of the condenser as shown below. Place the new long focal length lens, f', in front of the object so that the object is just outside the focal length, f'. Adjust the projecting lens until an image is found on the large screen.

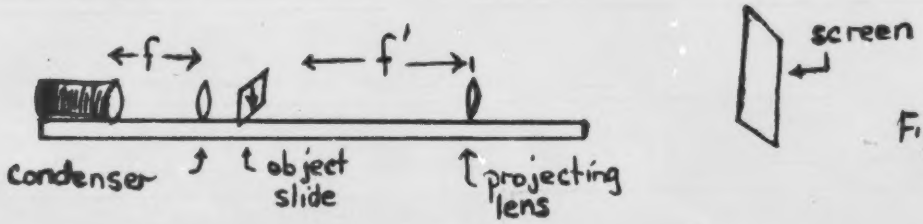


Fig. 5

This time assume a fixed image distance and use the lens equation to calculate what your object distance should be. Compare your calculated value to your measured value by computing the %-difference.

Practical note: 35 mm slide trays can replace the optical bench, and suitable lenses and screens placed in 35 mm cardboard mounts, which can then be placed at the correct distance in the tray. Such an arrangement has the advantage that it can easily be held up to look through - as a telescope. The Edmund company sells a complete kit of this nature.

35.9

The Camera & the Projector

Notes to instructor:

For the simple camera to show at the beginning of the lab, an old fashioned bellows camera is perfect. Try to pick up, say, a Speed Graphic at a surplus house even if you have to put your own lens in it. You can purchase a "Simple Experimental Camera" from Welch for $17.50 (catalog # 3744A) or make a comparable one.

It is suggested that you use regular 35 mm slides as objects rather than arrows, etc. The slides can be of "fun" scenes that you have taken around campus or scenic slides from the Grand Canyon, etc.

You may wish to require ray diagrams to go along with the observations or calculations.

You may wish to include what happens when an object is placed at the focal plane and within the focal point.

You may wish to acquaint the students with a pinhole camera, perhaps having them taken some pictures with it by time exposure.

Mirrors and Symmetry

Introduction. The following set of four experiments are intended to demonstrate the properties of reflections and other symmetries. Applications to two-dimensional patterns (friezes, border designs, mosaics, wall paper patterns, etc.) are emphasized. The experiments could complement lectures on various physical instances of symmetry and parity (e.g. crystallography and elementary particle theory). Experiment I is geometric in nature and its concepts are used in what follows. The other three experiments are essentially independent. Some mathematical concepts (groups, vectors) may not be familiar to the student and are included in a separate appendix. Finally, a short but useful and interesting bibliography is given. Suggested exercises are lettered in each part.

I. Isometries.

Equipment. One thin semireflecting surface (a 4" × 4" × 1 mm. glass plate will do) supported by two small stands to keep it vertical (small pieces of wood with a 1/2" × 1 mm. groove cut in each can be placed on the bottom of the two vertical edges of each plate). Protractor. Compass. Thin cardboard.

Discussion. 1. An isometry (or rigid motion) of the plane is a mapping or transformation which assigns to every point in the plane a unique point (called its image). An isometry preserves distance so that the distance measured between any pair of points is the same as the distance between their images. Isometries can be composed by first mapping by the first isometry and then mapping (the image) by the second isometry. This composition produces an isometry. The identity isometry leaves every point fixed.

(a) Show by examples using a compass (which can be used as in high-school geometry to draw equal distances) that when the images P' and Q' of two distinct points P and Q of an isometry are given, this uniquely determines the image R' of any other point R on the line joining P and Q and that R' lies on the line joining P' and Q'.

(b) If another point S does not lie on the line joining P and Q show that there are precisely two possibilities for the image S' of S. We will see that there are (many) isometries which can take a given point P to some other given point P'.

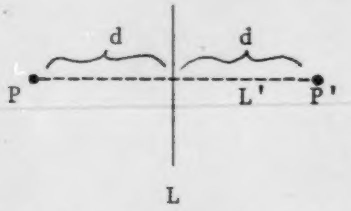
(c) If an isometry takes P to P', what are the possibilities for the image Q' of another point Q ? (Hint: Use your compass and the distance preserving property of isometries to answer the above three questions)

(d) Show that if we know the images of three distinct points (not on a line) P, Q, and R then we know the image of any other point.

This shows that an isometry is determined once we know what it does to (the three vertices of) a triangle. By laws of Euclidean congruence we may consider isometries as the sliding or flipping over and sliding of a triangle. Thus the image of a triangle with vertices A, B, C is a congruent triangle (one which may be superimposed with the original triangle either flipped over or unflipped). Cut a small triangle out of this cardboard and label one side D and its vertices A, B, and C in clockwise order. Turn the triangle over and label the other side O. Label the vertices on this side the same as you did on the other side (so that A, B, C will now read counterclockwise).

2. An example of an isometry is a reflection (through a line L). A reflection through L can be realized by mapping each point P through L to P' along a line L' containing P and perpendicular to L such that

The distance d (along L') from P to the intersection of L and L' is the same as the distance from that intersection to P':



(a) What is the image of a point on L?

A reflection through L can be performed by placing a thin vertical semi-reflecting (glass) surface along L. It should be thin to decrease the ambiguity of the reflecting plane. Look at the glass on the same side of a point P until you can see the reflected image of P in the glass. Then look through the glass and put the image P' on the other side of L at the point where you see the reflected image of P in the glass. In fact the image of any pattern you can see reflected in the glass may be transferred by the same method.

(b) Reflect your name through L.

(Note: The vertical glass is a useful geometric tool in general. Normals to curves through a point P may be found by adjusting the glass so that the reflection of a small portion of the curve on one side of P is superimposed on a small portion of the curve on the other side of P. Then the normal through P can be drawn by tracing along an edge of the glass. The tangent at P can then be drawn.)

(c) Find the images of two points P and Q when reflected through L by the method outlined above and by measuring distances verify that reflection is an isometry.

(d) What is the inverse of a reflection through L?

2. Draw two parallel lines L and L'. They will be parallel if they both make right angles with a third line L". The distance between L and L' is then defined as the distance between their respective intersections with L". Take a point P and find its image P' reflected through L. Then find the image P" when P' is reflected through L'. Perform the above composition of reflections with another point Q on the other side of L from P. Draw the line segments PP" and QQ".

(a) What angles do those segments make with L?

(b) How does the direction from P to P" (and Q to Q") compare with the direction from L to L'?

(c) Compare the three distances from P to P"; from Q to Q"; and from L to L'.

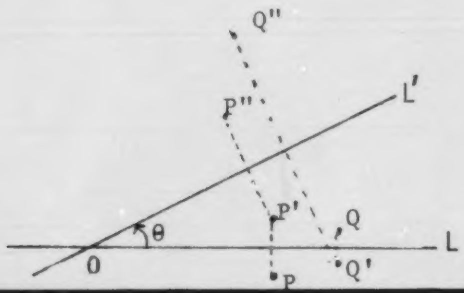
(d) How does the vector $\overrightarrow{PP''}$ compare with the vector $\overrightarrow{QQ''}$.

This type of isometry (the composite of two reflections through parallel lines) is called a translation and, as you may conclude from question (d) above is characterized by transforming each point by a fixed vector v.

(e) Trace your triangle ABC and its two images obtained by reflecting through L and then the image through L'. Describe directly how the image of the triangle has moved to its final image.

(f) Compare the translation constructed above with the one you get by first reflecting through L' and then through L.

3. In this part we will assume all angles are measured in a counterclockwise direction. Draw two lines L and L' which intersect at a point



O. Measure the angle θ from L to L' with your protractor. Take two new points P and Q on opposite sides of L and as in part 2 reflect them first in L and then (their images) in L' obtaining P" and Q". Measure the distance of line segment OP and the distance OP". Do the same with distances OQ and OQ". Measure the angle θ' between OP and OP". Measure the angle θ'' between OQ and OQ".

(a) Compare the four distances you measured and the three angles.

(b) Compare the distance PQ with the distance P"Q".

A rotation about a point O through an angle α is the mapping which preserves distances to O (i.e. O is fixed by the isometry) and which maps each point around O an angular distance of α. If you were accurate in (a) and (b) above you should be able to conclude that rotations are isometries and are given by the composite of two reflections through intersecting lines.

(c) What is the inverse of a rotation about O through an angle α?

(d) What is the composite of two rotations both about O through angles θ_1 and θ_2 respectively?

Two lines in the plane can either coincide, be parallel, or intersect (in a point). By the above two parts we may now conclude that the composite of two reflections is therefore either the identity, a translation, or a rotation (respectively).

4. Take your triangle ABC and trace it labeling the three vertices on the paper. Slide (without flipping) the triangle to a new position and trace ABC in its new position labeling those vertices A', B', and C' respectively. We know that this motion defines a unique isometry. Place the vertical glass midway between A and A' perpendicular to the line AA' (or on the point A if A = A'). If this is done accurately the glass should lie on a line L which reflects A to A'. Using the vertical glass, reflect ABC through L labeling the new vertices $A_1 = A'$, B_1, C_1.

Now do the same procedure reflecting B_1 into B'. (If they are already identical reflect through line A'B'.)

(a) Describe what has now happened to vertex A_1? To vertex C_1?

(b) Using parts 2 and 3 and (a) above explain why the result of sliding a triangle is either a translation or a rotation.

5. Starting with triangle ABC reflect it in some line, labeling the vertices of its image A_1, B_1, and C_1 appropriately. Now reflect triangle $A_1B_1C_1$ in some other line labeling this new image $A_2B_2C_2$. Do this a few more times. Record for each image whether the vertices A,B,C go around clockwise or counterclockwise. We know from parts 1 and 2 that the images are all congruent to ABC. If the vertices go around clockwise we say the image has the same orientation as ABC. Otherwise the vertices go around counterclockwise and the image has opposite orientation.

(a) Describe the images $A_1B_1C_1$, $A_2B_2C_2$, etc. in terms of whether the isometry composed of the above reflections could have been performed by sliding ABC (keeping its D side up) or by flipping ABC (to its O side) and then sliding.

(b) In general what is the case after an odd number of reflections? After an even number of reflections?

An isometry is direct if it preserves the orientation of a triangle. It is opposite if it reverses the triangle's orientation.

(c) Which of the above compositions are direct and which opposite?

Now trace ABC on the paper, flip it over to its O side and slide it to a new position labeling the respective vertices A', B', and C'. Perform the same two reflections that you did in part 4 reflecting A into A' obtaining vertices $A' = A_1$, B_1, and C_1. Then reflect B_1 into vertex B', obtaining triangle $A_2B_2C_2$.

(d) Does vertex A_2 coincide with A'? Does vertex C_2 coincide with vertex C'?

Reflect $A_2B_2C_2$ in line A_2B_2 obtaining $A_3B_3C_3$. If you did this correctly, $A_3B_3C_3$ should be identical with A'B'C'.

(e) Why can it be said that any isometry in the plane can be realized as the composition of zero, one, two, or three reflections? Which of these are direct and which opposite?

The isometry which needs no reflections is of course the identity while those which need one are the opposites.

(f) What do we call the isometries which need two reflections?

The isometries which need three reflections can also be performed by a translation along a vector v followed by a reflection in a line parallel to v. These isometries are called glide reflections. For example, on a typewritten line, the letter "b" can be taken into the letter "p" to the right of it by a horizontal glide reflection.

In conclusion we have shown that there are four types of isometries (in addition to the identity): two direct (rotations and translations) and two opposite (reflections and glide reflections). Isometries may be composed and each has a unique inverse. They form a group (see the appendix).

II. Reflections in Space.

Equipment. A mirror (as large as possible). Two hinged mirrors (1' × 1' mirrors connected back to back by a cloth backed tape so that they may be opened from 0° to 180°). Protractor. Transparencies (e.g. for an overhead projector).

Discussion. Isometries in space are also (spatial) distance preserving mappings. Reflections through a plane (mirror) are a particular example.

1. Look into a mirror. Wink your left eye, etc.

(a) Describe as best you can what you are seeing.

Write something on a piece of paper and reflect what you have written.

(b) Describe this image. Does the mirror reverse the directions up and down? right and left?

Tilt your head 90^0 to one side still looking into the mirror.

(c) Would you now say the mirror reverses up and down? right and left?

Write something on the transparency. Hold the transparency in front of you and compare the view that you have of it (through the transparency) with the mirror image which you see.

(d) Describe again what directions are reversed.

Assume the mirror is against the north wall of the room. Label the other walls east, south, and west appropriately. Point up and observe which way your image points. Do the same for the five other directions (down, N, E, S, and W). Point in other directions such as NE.

(e) Record how the mirror changes each of these directions.

(f) What are the apparent coordinates of the image of a point with coordinates (x,y,z) where x is the distance into the room from the north wall (so that negative values correspond to points beyond the wall), y is the distance into the room from the west wall and z is the distance from the floor.

Write the word "Physics" on a piece of paper. Reflect it in the mirror. Now take the piece of paper and tape it to the mirror so you can see the word.

(g) Record the difference between the reflected word and the taped word.

Note that when you taped the paper onto the mirror you had to turn the paper around in order to see the word. How did you turn the sheet? How else could you have turned the sheet to make the other side visible?

(h) Why is it more natural to think of half-turn rotations in space made through a vertical axis rather than some other axis (like a merry-go-round instead of a ferris wheel for example) so that top and bottom remain in the same position but front-back and left-right relationships are reversed?

Think about the phenomenon of apparent left-right reversal in a mirror by the psychological preference for comparing a mirror image with the result of "walking through the mirror" and doing an about face.

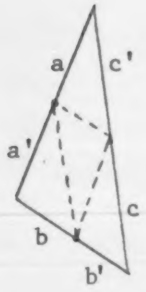
(i) How would you describe mirror reversals when compared with walking through the mirror and turning around by doing a headstand.

It is fair to conclude that what mirrors in fact "reverse" are directions perpendicular to the plane of the mirror, so that they reverse front-back but they reverse neither top-bottom nor left-right relationships.

2. Make a triangle and label its vertices. Note that in space there is no problem in "flipping" a triangle since we are not restricted to a plane (where we would need to take the triangle out of the plane to turn it over). It is appropriate to imagine the triangle as having no thickness so that labels on the vertices should be visible on both "sides" of the triangle.

(a) Can the triangle be superimposed in space with its mirror image?

Now make two tetrahedra which may be superimposed by cutting out two congruent scalene triangles (with the same orientation) and folding up along the three lines which connect midpoints of pairs of adjacent edges, taping a to a', b to b', and c to c' in the following picture:



Label the four vertices of one and label the vertices of the other in the same way so that the tetrahedra can be superimposed. Reflect one tetrahedron in the mirror. (You may not be able to see all four vertex labels, but you can see three and know what the fourth label must be.)

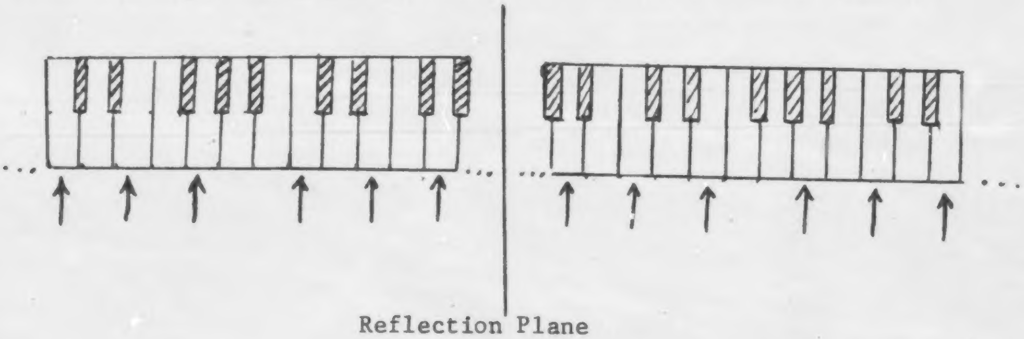
(b) Can one tetrahedron be superimposed on the mirror image of the other?

Take apart one of the tetrahedra and reconnect the same edges of the triangle but fold in the other direction (i.e. down or start with the triangles oppositely oriented before folding up their respective sides).

(c) Now can one tetrahedron be superimposed on the mirror image of the other?

Two shapes in space are termed congruent if one can be superimposed on the other or on the other's reflected image. In the former case (as in the plane) they are said to have the same orientation or parity and in the latter case, opposite orientation. This distinction is very important in the theory of particle physics. Reference [7] is a very readable account of physical parity; [9] is rather more technical. Similar distinctions make it hard to put a left-handed glove on your right hand (unless you turn it inside out) while the notion of a right-handed monkey wrench is the basis for a mildly amusing joke.

(d) Make up and perform your own experiments with parity. For example, students with a musical background can imagine reflecting the keyboard of a piano in the mirror. When this is done what do the notes of a reflected C major chord look like (i.e. when you play those notes, what chord does it look like the hand in the reflection is playing as in the diagram below)? Answer these questions for major 7th and minor 7th, diminished 7th, and augmented 5th chords. Note that if, for example, one major chord is reflected to a minor chord then every major chord is reflected to a minor chord. You should be able to understand why this is true from Experiment I above, since any major chord can be viewed as a translation of another.



3. Place the hinged mirror on the table with the hinge vertical and observe what happens to the multiple images of your face as you increase and decrease the angle between the mirrors. For example, wink your left eye and observe where that eye appears on each image. Using your protractor make a 90° angle between the mirrors. You should now be able to see your image in the combined mirrors with the hinge vertically bisecting your face.

(a) What directions are now reversed? Is your image of the same or opposite parity as yourself? Using the ideas in Experiment I explain the space isometry which transforms you to your image.

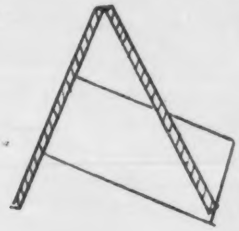
Now measure an angle of 60° between the mirrors.

(b) What do you observe?

Place some objects randomly on the table between the mirrors.

(c) Describe how to make a kaleidoscope. As you see the objects reflected in images which seem to circle around the hinge describe which images are direct and which are opposite. What motion in the plane of the table have produced each of the images you see?

Anchor one of the mirrors and place a (rectangular) sheet of paper along (or draw a perpendicular line to) the side of that edge. From the top, the set up should look like the following:



Rotating the other mirror, observe the images of the sheet (actually you only need to observe one edge of the sheet perpendicular to the mirror).

(d) Describe how to make an angle of 60° using the hinged mirrors (without the protractor).

(e) How is an angle of 45° obtained? Check your answers with a protractor.

(f) What other angles can you find in a similar way?

(g) If you wanted to make a kaleidoscope which showed exactly four other direct images of the region between the mirrors how large would you have to make the angle? How many opposite images would there then be?

(h) Does moving your head around and viewing the inside of the mirrors from different positions affect your result?

III. Symmetries

Equipment. Xerox copies (or the original illustration) in references [1, 3, 5, 6, 8, and 10] or other examples of symmetric patterns. Vertical semireflecting mirror (as in Experiment I) or standard square mirror. Facilities to make transparencies (e.g. Xerox 4000 copier) of the illustrations in the above references. (Optional: Two identical 35 mm. slides of the above illustrations and two slide projectors.)

Discussion. 1. For our purposes, a pattern (of the plane) is obtained when some of the points are colored differently from others (this is a rather fancy definition of "picture" or "design" but the concept of when two points are in regions of the same color will be an important one). Allowing one color to denote those points not otherwise colored, we note that any figure such as a drawn circle is a pattern. A symmetry of that pattern is an isometry which preserves the pattern in that points are mapped to points of the same color. It is easy to see that the symmetries of any pattern form a subgroup of the group of all isometries called the symmetry group of the pattern. In fact the group of all isometries is the symmetry group of the (uncolored or uniformly colored) plane itself. It is a consequence of Experiment I that the three vertices of a scalene triangle (or the triangle itself) viewed as a pattern has a symmetry group consisting of only the identity. We call such patterns (or figures) asymmetric.

(a) Describe all the symmetries of a point (i.e. one point colored differently from the other points in the plane).

(b) Describe all the symmetries of a line. Hint: Mentally review the symmetries (reflections, rotations, translations, and glide reflections) and determine which of these takes the point in (a) into itself or points on the line in (b) into other points on that line.

(c) What are the symmetries of a regular polygon (equal sides and angles)? a rectangle (which is not a square)? a circle?

(d) Why can't a translation or glide reflection be a symmetry of a finite pattern (i.e. one which does not extend indefinitely)?

If a pattern has at least one reflection in its symmetry group it is said to have reflective symmetry. If it has a rotation it is said to have rotational symmetry.

(e) For the capital letters (in the Doric style, without seraphs) which have rotational and which have reflectional symmetry? Which seven letters are asymmetric?

Every symmetry group of order two contains the identity and one other isometry (which must therefore be its own inverse).

There are two quite different symmetry groups of order two: one containing a reflection (and the identity) and one containing a rotation of 180° (a half-turn).

Three predominant themes in art are the frontal view human figure, the (Latin) cross, and the Yin-Yang symbol (as on the Korean flag).

(f) What are their symmetry groups?

(g) A swastika has what kind of symmetry? What is the order of its symmetry group? Generalize this figure to a "three-branch swastika" (such as the arms of the Isle of Man, the "legs of Man") a "Five-branch swastika", etc. to obtain a pattern with only rotational symmetry whose symmetry group has any specified order.

2. A discrete (or non infinitessimal) symmetry group is one which has a shortest translation (i.e. some distance such that no translation is through a shorter distance). In addition, for any point P, the subgroup which maps P into itself must be finite. In other words, there is a

smallest angle for rotation. Note that the symmetry group itself need not be finite. For example, the symmetries of an infinitely long sequence of dashes: ---------- has an infinite number of translations (and an infinite number of reflections) but one shortest translation (the distance from, say, the middle of one dash to the middle of its neighbor).

For the remainder of this experiment all symmetry groups will be discrete. We will be rather imprecise about when two different symmetry groups (or their patterns) are "essentially different". This should cause no undue problem once we have experimented with various patterns and become more familiar with their symmetries. It is our goal to distinguish between different patterns and recognize similar ones. We will start with patterns which are subsets of a line (the simplest case) and go on to those which are subsets of a strip and finally treat the general planar case.

Here are some general criteria to apply when classifying patterns:

When two translations have different directions, their respective lengths and the angle between them is not important in deciding if they are essentially different patterns.

However, the (smallest) angle of a rotation (about a point P) is important as this determines the number of possible rotations about P. For example, if that angle is 60° there are exactly six rotations about P (including the identity). Here are some other considerations which will lead to distinguishing patterns.

Which types of symmetries are present (e.g. are there glide reflections, etc.).

What is the largest order of a subgroup which fixes some point of the pattern.

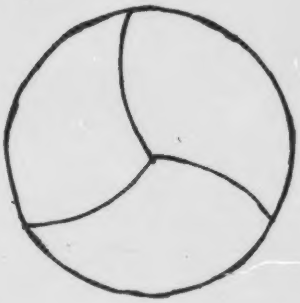
How the directions of the translations interact with lines of reflections or directions of glide reflections. Similarly, do reflection lines go through centers of rotation?

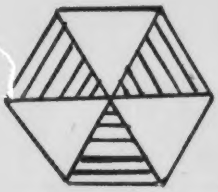
Whether the pattern contains glide reflections without containing the translation (glide) from which it was formed.

Briefly, it is the interaction of the symmetries of a pattern (their group structure) which is more important than the pattern itself.

(a) As we have seen, only rotations and reflections can fix a point. Using the results of Experiment I show that these are essentially two different patterns with a smallest rotation of 360°/n for each value of n. We will call one pattern the n-pronged swastika pattern and the other the regular n-gon pattern (where a 2-gon can be though of as a line or a rectangle). Which pattern is similar to a star?

(b) Which patterns are similar to the following:





3. There are only two patterns on a line: the Morse code message consisting of a's (dot dash space dot dash space etc.); and the Morse code message consisting of e's (dot space dot space, etc.).

(a) Why are they essentially different patterns?
Note that on a line
we cannot distinguish between a reflection (though a line perpendicular to the line) and a half turn (about the point of intersection) although in the plane we saw that rotations and reflections are quite different.

(b) Which pattern (a or e) is that formed by an equal spacing of dots which alternate in colors red blue blue red? red blue blue? red white blue?

4. The patterns encountered in the remaining parts of this experiment will be a bit more difficult to analyze. To aid in their analysis, the following procedures can be followed. The pattern can be Xeroxed and a transparency copied from the same original. Reflections can be detected by a vertical mirror or by the vertical semireflecting surface as in Experiment I. Reflection lines can then be labeled. Rotations and translations can be detected by using the transparency and sliding it along the original until the patterns line up. We then observe that unless the transparency is slid keeping its sides parallel to the original, a fixed point will occur, thereby indicating a rotation. Glide reflections can be detected by flipping the transparency and moving it around to see if the patterns can then be made to coincide. An alternate method (more useful for demonstration purposes than for individual experimentation) is to focus two slide projectors on a common screen and to superimpose their images after one of the projectors has been rotated, translated, etc. Reflections and glide reflections can be discovered by following the same procedure after one of the slides has been flipped.

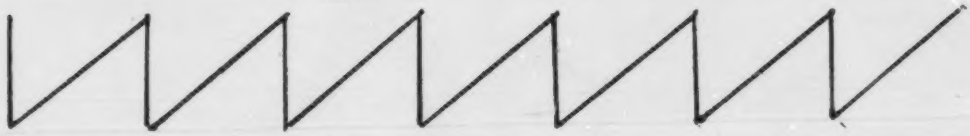
(a) What symmetries must a pattern have if it doesn't matter how a 2" x 2" slide of that pattern is put into the projector? How many different ways can it be put in?

We will now explore the strip patterns (or frieze patterns). These are patterns which may be placed on an (infinitely long) strip. They all contain translations but these translations are only in one direction (parallel to the length of the strip). A concept which is important is that of a fundamental region of a pattern. This is a smallest region which contains the basic character (design elements) of the pattern, as the pattern is formed from a sequence of nonoverlapping translations of this region. Fundamental regions are not unique. For the a pattern in part 3, the pattern "dot dash space" (·-) is a fundamental region.

(a) What other fundamental regions generate the a pattern?

There are seven basic types of strip patterns. We will name them by using the mnemonic device of using words of (lower-case) letters which form their fundamental regions. For example, the pattern named "pd" is the pattern formed by the infinite sequence ...pdpdpdpd...

The pd pattern is exemplified by the pattern:



The seven patterns are:

p
c
pq
pb
pd
x
pdbq

(a) Explain why pb is different from pd. Why is pdbq different from each of the others?

(b) Name the pattern formed from a string of (capital) N's? M's? O's? And a zigzag?

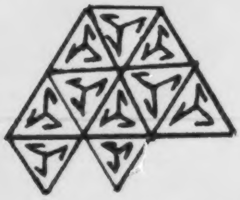
(c) Classify some selected borders of Greek vases in reference [3] and find a fundamental region for each. Can you find all seven strip patterns?

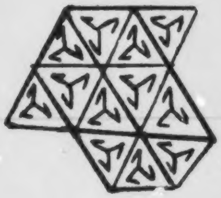
5. There are 17 wallpaper patterns (i.e. patterns in which a finite fundamental region can fill up the entire plane). All these patterns are characterized in that there is a shortest translation and a next shortest translation in a different (nonparallel) direction. All other translations

are compositions of these translations (and their inverses). A parallelogram with adjacent sides formed from these vectors gives a fundamental region although other shapes are possible (e.g. in Escher's work). This is the first time we will encounter rotations (other than half-turns) along with translations. By the very important "crystallographic restriction " the only smallest rotations which are possible are through angles of 180°, 120°, 90°, and 60°. This restriction is inherited by the crystallographic space groups. In Experiment IV we will gain some insight why these are the only angles available.

Again we will name the seventeen patterns mnemonically but we refer the student to [2] for a more esthetic display of these patterns (we use the same order as [2]) and to [4] for their crystallographic names. The patterns are grouped as to smallest rotation.

Recall that there are three tessellations (tilings) of the plane with regular polygons: triangles (as on a geodesic dome); squares (a checkerboard) and hexagons (a honeycomb). They constitute a starting point for the last eight wallpaper patterns. In particular, if we fill each polygon (for example triangle) with (three) pronged swastikas going the same way, we say we have the oriented (triangle) pattern. If the swastikas alternate in clockwise—counterclockwise directions we say we have a doubly oriented unoriented (triangle) pattern. Without swastikas we have the triangle pattern. The first two cases are illustrated below:





We may apply the same principle to rectangles (and two-pronged swastikas) to get patterns five through seven.

Here is the list:

No rotations:	p	(i.e. a parallelogram containing p is used to tile the plane).
	OT TO	(a square containing OT is used, however the fundamental region is simply TO).
(horizontal)	pq	(or Y).
(vertical)	p q .	
180° rotations:	oriented rectangles.	
	doubly oriented rectangles (the "brick pattern").	
	(unoriented) rectangles $\binom{pq}{bd}$.	
	pd bq .	
	pb qd .	
120° rotations:	oriented triangles (we must not allow 60° rotations through points where 6 triangles meet so imagine the triangles are alternately colored with two colors).	
	(unoriented) triangles (in two colors as above).	
	doubly oriented triangles.	
90° rotations:	oriented squares.	
	(unoriented) squares.	
	doubly oriented squares (the "overweave-underweave" pattern).	
60° rotations:	oriented hexagons.	
	(unoriented) hexagons.	

(a) Why don't we have a doubly oriented hexagon pattern?

(b) What distinguishes the three different patterns with 90° rotations?

(c) Which is the pattern formed from an infinite two-colored checkerboard?

(d) Classify some selected patterns in [5], [6], and [8] and find a fundamental region for each.

(e) Make up your own pattern for each of the seventeen classes.

IV. Reflection Groups

Equipment. At least sixteen 1' × 1' mirrors. One mirror 17" × 1'. Glass cutter. (If it is not feasible to cut mirrors yourself, have 10 1' × 1' mirrors, four rectangular mirrors of length 1' and widths 6", 10.4", 8 1/2" and 8 1/2" respectively; also triangles formed from a diagonally cut 1' × 1' mirror and a 1' × 17" (rectangular) mirror. Styrofoam cube (with 6" side). Extra mirrors. Heavy duty clothbacked tape at least 1" wide.

Discussion. 1. Among the symmetry groups we explored in Experiment III, some of them are generated by reflections. That is, every symmetry can be realized as a composition of reflections. A symmetry group is a reflection group generated by a set R of reflections if every symmetry is a composition of elements in R. Equivalently, this means that the only subgroup of the symmetry group which contains the reflections R is the whole symmetry group itself. In terms of patterns, this means that if we take that part of the pattern bounded by the reflection lines associated with R, and look at that part of the pattern and its reflections in mirrors placed on those lines, the whole pattern is visible.

Two simple cases of reflection groups have already been observed. One of them is the familiar "barber shop pattern" as you see yourself multiply reflected in two parallel mirrors.

(a) Which of the seven strip patterns in Experiment III, part 4 is the barber shop pattern? Which Morse code pattern is it?

The fundamental region (as described in Experiment III) of a reflection group is not the region between the mirrors (in the barber shop pattern it is twice as big) since the fundamental region gives the pattern by translations only. We will call this smaller region bounded by the mirrors the fundamental reflection region.

Another example of a reflection group is the symmetry group of a regular polygon (Experiment III, part 2).

(b) What is the fundamental reflection region of such a pattern.

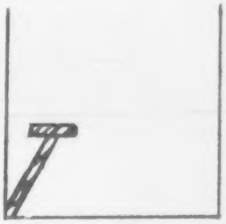
Evidently, two reflections can generate only the barber shop pattern or the symmetry group of a regular polygon. To create a wallpaper pattern at least three mirrors are needed.

When three mirrors are placed in a triangle in which one of the angles is not 30°, 45°, 60°, 90°, 120°, or 180°, then it will generate a rotation which is not one of the rotations possible in the crystallographic condition of Experiment III.

(c) Form a triangle with an angle different from one of those specified above and observe the pattern formed. Why is it not a discrete symmetry group?

2. Using a one foot mirror as a unit, we will create all the symmetry groups in Experiment III which are reflection groups. Our method will be to cut mirrored squares to the proper specifications and then connect their external sides with tape.

Beside the "barber shop pattern" only one other strip pattern is a reflection group. This is the x pattern. Take three mirror squares and connect the three to form three sides of a rectangle (or square). Look into the pattern this forms. A fundamental reflection region is then one of the four "legs" of the x. Looking from above we have the following picture:



Of the seventeen wallpaper groups, four are reflection groups. Since at least three mirrors are necessary at least two of them must intersect and therefore the pattern must have some rotational symmetry.

(a) Why must there be rotational symmetry?
There is one reflection group whose minimal rotation is 180°. By Experiment I this means that two mirrors must meet at a 90° angle. In fact all angles between adjacent mirrors must be 90°.

Make an open rectangular (or square) box with four mirrored sides (so that they all reflect into the middle of the box).

(b) Which pattern did you create? What part of its mnemonic name forms a fundamental reflection region?

(c) Create a design and sketch the pattern created when that design is placed in the middle of the mirrors.

Construct the following internally reflecting mirrored triangles (actually open prisms with a triangular base):

an equilateral triangle (with three equal squares and angles of 60°, 60°, 60°)

a right isosceles triangle (with mirrors of height 1',; one of width 1'; and two others of width $\sqrt{2}/2 \approx 8\ 1/2''$ respectively so that its angles are 90°, 45°, 45°).

half an equilateral triangle (with widths 12", 6", and $\sqrt{3}/2 \approx 10.4''$ and angles 30°, 60°, 90°).

Observe as you did before the three patterns formed.

(d) In these three cases what pattern did you create? For each how do fundamental reflection regions connect into one fundamental region?

(e) Draw fundamental reflection regions for the patterns in Experiment III which you classified as reflection groups.

3. In crystallography, the space groups (i.e. discrete symmetry groups of three-space) are studied. Whereas there are seventeen essentially different symmetries which fill up a plane, there are over 250 which fill up space. We can get some idea of their complexity with two examples of reflection groups.

The analogy of the unoriented rectangle pattern in three-space is the unoriented box pattern. (A pile of bricks with no overlapping). Make a five sided cube out of mirrored squares. (Put a mirrored bottom on the four mirror box you created in part 2.) Take a large mirror and place it on the top with a small opening so that you can see inside. Put a flashlight inside and observe this three-dimensional pattern. We remark here that this illusion was created on a mammoth scale at the "Labyrinth" exhibit of the Montreal 1967 Exposition. We will now observe the additional symmetries which result when we consider the symmetries of a cube (and not, as above the reflections which exist in a rectangular box). A cube has three pairs of opposite faces and six pairs of opposite edges. These give nine reflection planes of a cube (three of which interchange opposite faces and six of which interchange opposite edges).

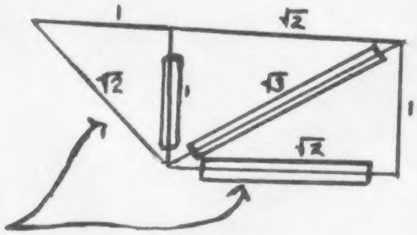
(a) Describe how those planes intersect the faces and edges of the cube. Where do they all intersect each other?

These reflections and their compositions generate 48 distinct isometries which take the cube into itself. 24 of them are direct in that they can be realized by rotating the cube (and preserving parity). Note that any one of six square faces can be "on top" and then any of its four adjacent faces can "face front". These reflections and their interactions can be best visualized by making a few cuts in a styrofoam cube (with a band saw) along reflection planes. The student should note the similarity between symmetries in three-space of a cube and the symmetries (in the plane) of a regular polygon. In both cases half of them can be realized by rotations. This is true in general. When all the cuts have been made the cube is cut up into 48 congruent regions, each of which is a fundamental region of the cube.

(a) Show that a fundamental region of the cube is a four-sided pyramid (tetrahedron). Its base is a 45°, 45°, 90° triangle which is formed by 1/8 of one of the bounding planes of the cube and the vertex of of the tetrahedron is the center of the cube.

When we make the three sides of the pyramid out of mirrors we will be able to peer inside and see the cube with all of its reflecting planes. Since many reflections are occuring, the mirrors must be cut quite accurately to observe the proper effect. Cut a 1' × 1' square in half diagonally for one side. The other two sides are formed from (directly) congruent triangles of sides $1, \sqrt{2}, \sqrt{3}$. We can make both of these simultaneously if we diagonally cut a mirrored rectangle of dimensions 1' × 17" (more precisely $16\frac{31}{32}''$).

Taping these three triangles together as below we obtain our fundamental region:



tape these
edges together also

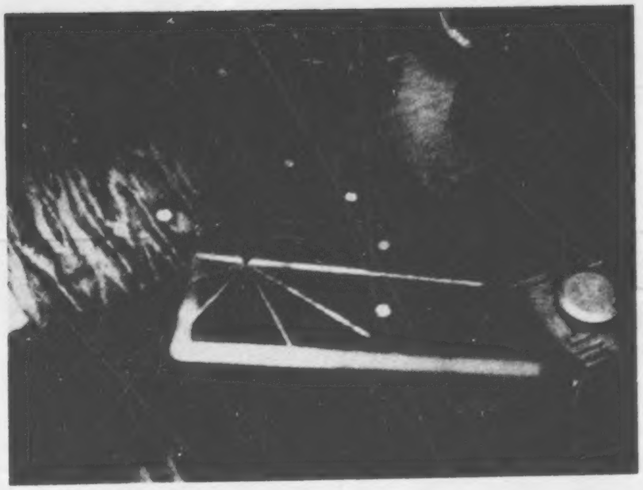
(b) Explain what happens when this region is seen through its open base. In particular, trace a light ray which appears to be reflected off the rear face of the cube.

When the fourth side is partially covered with a mirror as in the case of the mirrored box, another space group is seen.

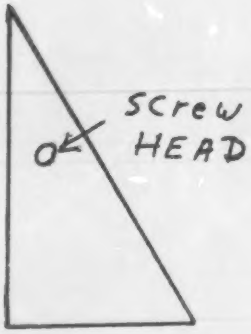
(c) What is the analogous wallpaper group?

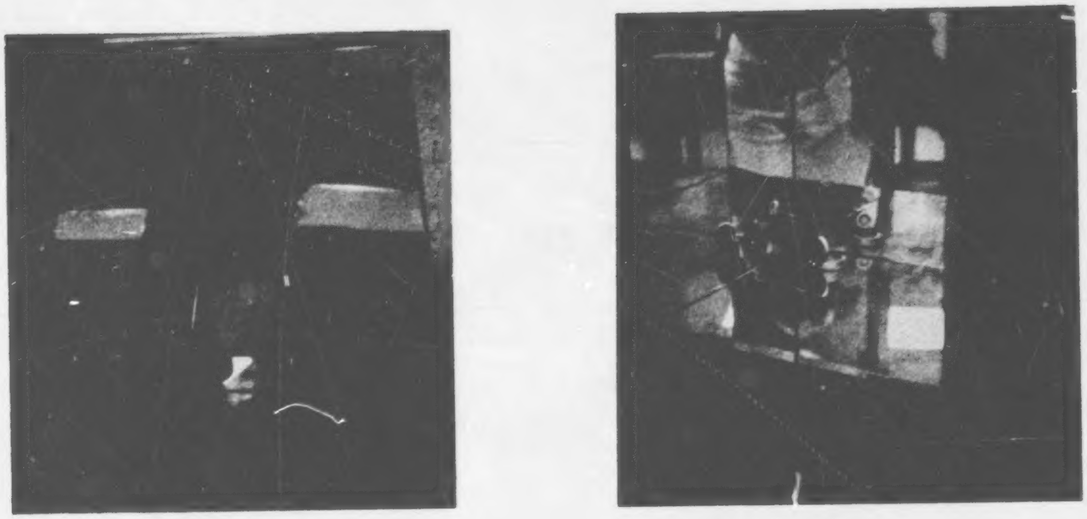
The more ambitious student can perform this with a regular tetrahedron (bounded by equilateral triangles).

(d) How may reflection planes will there be? How many fundamental reflection regions? How many ways are there to rotate a tetrahedron in three space? (The other half of the isometries correspond to rotating the tetrahedron's mirror image as in Experiment II).



30°-90°-60°
← Triangle

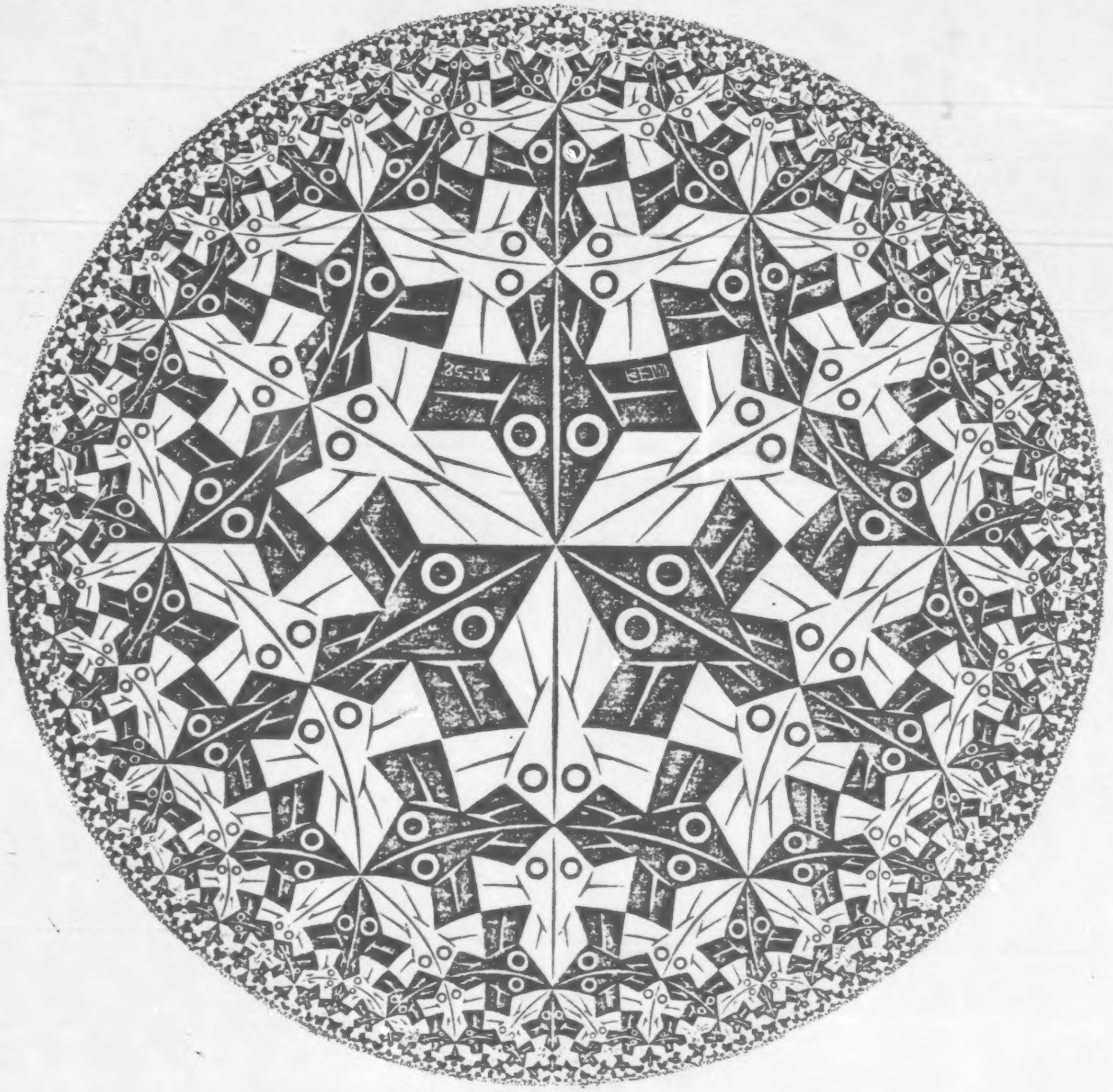




TWO VIEWS OF THE MIRROR CUBE

OVER PAGE
ESCHER PRINT, TO BE USED WITH MIRRORS TO SHOW
SYMMETRY PROPERTIES

36 4.7



22. **Cirkellimiet I - Circle limit I - Kreislimit I - Limite circulaire I - Cirkelformad begränsning I**

Appendix. A group is the mathematical term for a set of elements (e.g. real numbers) and a composition (addition in our example) which assigns to any (ordered) pair of given elements a third (not necessarily distinct) element (an addition table). In general (unlike addition) two elements will compose to give different elements depending upon in which order they are composed. The group must possess an identity element (zero) which when composed with any element in either order gives that number in the composition (0 + 3 = 3 + 0 = 3). Also every element must have a unique inverse (its negative) which composes with the element to give the identity (3 + (-3) = (-3) + 3 = 0).

Another property enjoyed by a group is that of associativity which in its simplest terms means that a string of compositions may be evaluated by composing any consecutive pair of terms thereby shortening the chain until eventually one element results (so that 1 + 2 + 3 can be evaluated as 3 + 3 = 6 or as 1 + 5 = 6). This means that any choice of consecutive terms may be chosen first so that unless otherwise specified one may as well compose "left to right" so that 1 + 2 + 3 + 4 = 3 + 3 + 4 = 6 + 4 = 10. A subgroup of a group is a subset of its elements which itself is a group using the same composition. An arbitrary subset of a group is a subgroup if it contains the identity, inverses of all its members, and the composition of any two of its members (including the composition of a member with itself).

The integers (...,-2, -1, 0, 1, 2,...) are a subgroup of the real numbers (using addition as composition). Other groups include the nonzero real numbers with multiplication as composition. The positive real numbers are then a subgroup.

Vectors in the plane are directed line segments (or "arrows") and two vectors are considered the same if they point in the same direction (i.e. they are parallel with their heads facing the same way) and have the same length. This gives a 1 - 1 correspondence between vectors and pairs of real numbers with the pair (x,y) corresponding to the vector whose tail is at the origin of the Cartesian plane and whose head has coordinates (x,y). Vectors form a group in which composition is vector addition. Vectors v and w may be added either by "head-to-tail" composition (i.e. superimposing the head of v with the tail of w and considering a new vector with v's tail and w's head) or by adding their first coordinates together and their second coordinates together. The identity of the group is the zero vector, the unique vector with zero length. The inverse of a vector is the parallel vector of the same length which faces in the opposite direction.

As a result of Experiment I, the isometries of the plane also form a group and the vectors can be thought of as a subgroup (of translations) of this group. Isometries and translations (vectors) in three-dimensional space also form groups.

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2. Caldwell, J. H., *Topics in Recreational Mathematics*, Cambridge, 1966.
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7. Gardner, M., *The Ambidextrons Universe*, Menter, 1969.
8. Torres Balbas, L., *La Alhombra Y El Generalite de Granada*, Editorial Plus Ulta (Madrid), 1953.
9. Wigner, E.P., *Symmetries and Reflections*, Indiana U. Press, 1967.
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ADJACENT COLORS

A number of books treating the phenomena of adjacent colors and various colored foregrounds and backgrounds are familiar to those interested in physiological optics (1, 2, 3, 4, 5).

More extensive treatment of certain effects are to be found in the works of some artists (6, 7, 8, 9). These books ordinarily contain somewhat better color plates than those mentioned in the first paragraph.

The observations outlined in Josef Albers "Interaction of Color" can be a student experiment (especially Chapters X through Y).

If desired the relatively inexpensive soft cover edition can be considered a piece of laboratory equipment. Some may wish to supplement this with material from references one through five or other sources.

If your institution has the kit Albers: Formulation, Articulation; it constitutes experiments in itself. However, the fact that it was printed in a limited edition and its 1973 price of $2,000 may bring up some obvious problems about using it for student labs.

REFERENCES

1. Clulow, Frederick W., Colour, Its Principles and Their Applications, 1972, London, Fountain Press ISBN 0-852-42098-6

2. Rainwater, Light and Color, Golden Press, Western Publ. Co.,-Paperback

3. Wilson, C. W., Seeing and Perceiving, 1966, Pergamon Press-Paperback

4. The Science of Color - Committee on Colorimity of the Optical Society of America, 1953

5.

6. Albers, Josef; Albers: Formulation, Articulation (LC-75-12577) Illust.

7. Albers, Josef; Interaction of Color: Unabridged Text and Selected Plates (abridged LC 74-147901) Illust. 1971, paper $4.95 Yale Univ. Press.

8. Itten, Johannes; The Art of Color (trans. by E. von Haagen) 1961, New York, Reinhold Publishing Company

9. Itten, Johannes; The Elements of Color (ed. by Faber Birren) 1970, New York, Van Nostrand Reinhold. This is based on Itten's The Elements of Color but is considerably briefer.

The Human Eye

Cenco has for some years sold an optical model (not physiological model) of the human eye. The associated experiment is instructive and popular with most students. In short the experiment goes somewhat like this. (Figure 1) The water filled tank represents the fluid filled eye and C is a lens let into the side to represent the cornea, a white circle painted on R represents the retina. (the fovea centralis and blind spot are marked).

A set of lenses is provided. By placing a lens (representing the crystaline lens) at L distant objects are brought to focus on R.

Accommodation is illustrated by bringing closer objects into focus by placing a lens of greater power at L.

Visual defects may be simulated by lengthening or shortening the "eyeball" by moving R forward or backward. These defects are corrected by placing a series of spectacle lenses at S and observing what type of lens corrects what type of defect.

Astigmatism is simulated by placing a cylinder lens at L and correcting the effect by rotating another placed at S.

Various combined defects may be simulated by the combination of lenses at L. Questions of varying degrees of sophistication may be asked.

In spite of its popularity the present price > $100.00 for the eye model and lenses and ~ $30.00 for the light source sometime prohibit its use because of the one-experiment-only nature eye model and lenses.

This note suggests how we can assemble an eye model from readily available materials, the total cost of which (aside from the contents of the tin cans) did not exceed $20.

Figure 2 represents a tin can of the type ordinarily containing boneless ham.

A hole for a corneal lens was let into the end using a chassis punch (drilling or snipping is O.K. but leaves less even edges). The lens used here was a plano convex lens The lens and its supporting tube were fixed in place using "5 minute" type epoxy. The general construction details are marked on the diagram.

The inside and outside of the can were painted with spray flat black enamel and the cornea with spray epoxy white enamel. They seem to adhere well to a cleaned can surface and have stood up well after being immersed in water.

The design criterion for the corneal lens is that it brings distant objects to a focus behind the rearmost position of the retina. Possibly even slightly outside the can.

Suitable lenses were obtained by calling an optometrist, explaining the educational nature of the project and having him obtain the lenses. Some (though probably not all) lens manufacturers are reluctant to sell to "lay people" .

The selection of a lens for the cornea is the only critical part. One may calculate the appropriate image-object distances using standard equations for summerged lenses (e.g., Strong, Concepts of Classical Optics). It is well however to roughly check the calculation by filling a tin can to the appropriate depth with water, hold the lens on the surface and observe the image of the sun or a high overhead lamp on the bottom of the can.

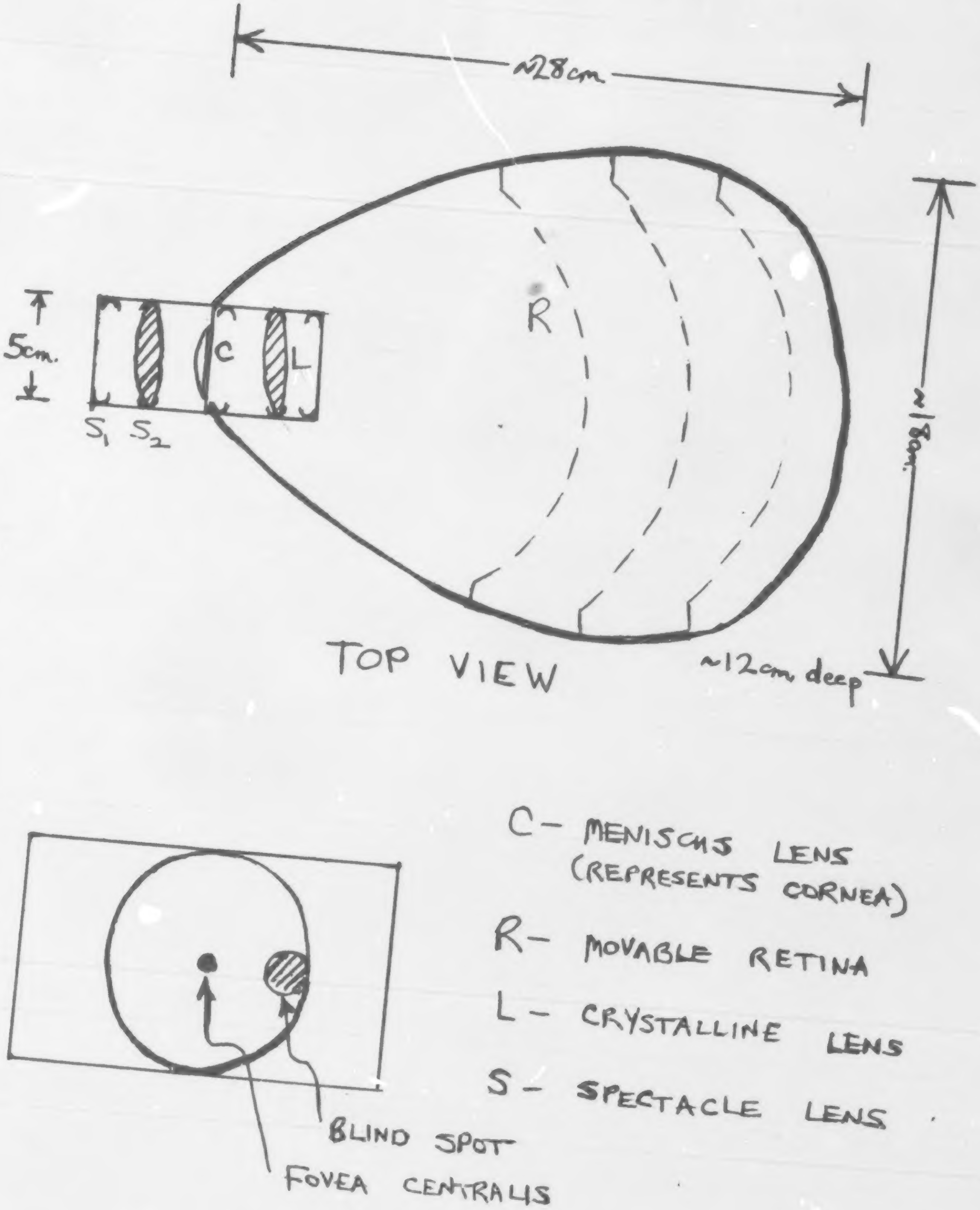
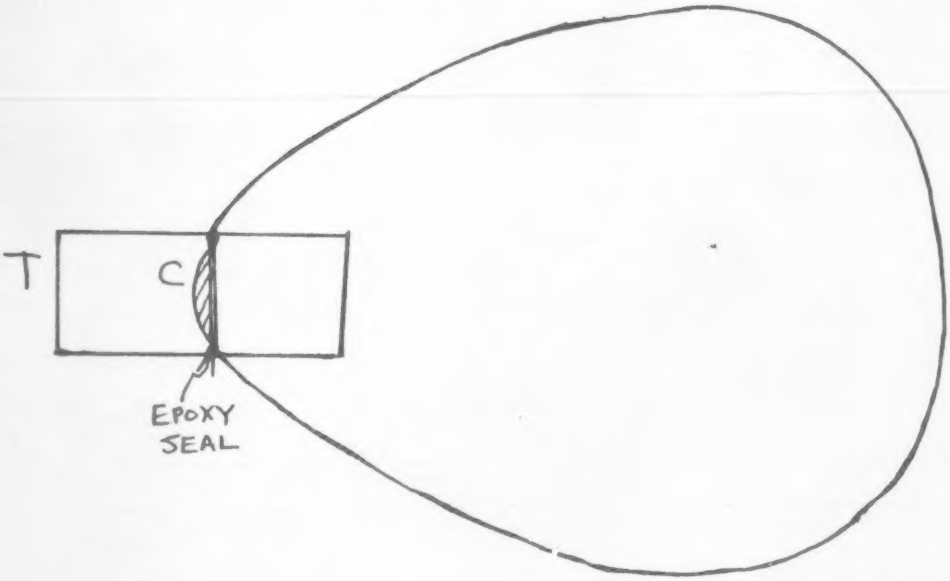

~28cm
5cm.
S1 S2
C
L
R
~18cm.
TOP VIEW
~12cm. deep
R
BLIND SPOT
FOVEA CENTRALIS
C- MENISCUS LENS (REPRESENTS CORNEA)
R- MOVABLE RETINA
L- CRYSTALLINE LENS
S- SPECTACLE LENS

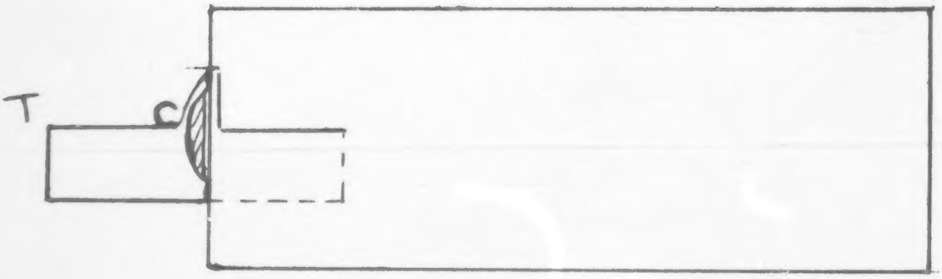
FIG. 1


T
C
EPOXY SEAL

TOP VIEW

T– SUPPORTING TUBE FOR L&S

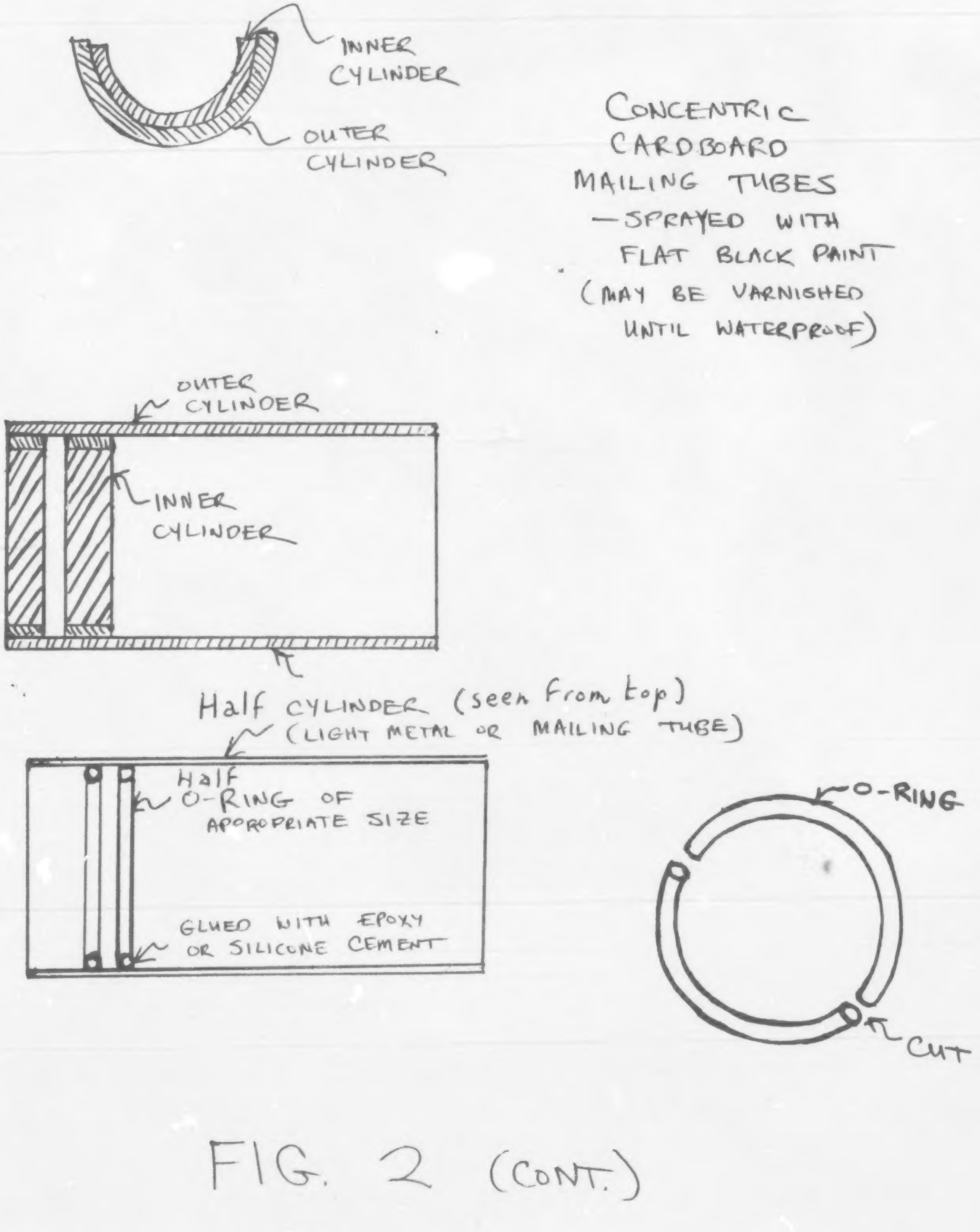
C– CORNEAL LENS



SIDE VIEW

FIG. 2

CONT. ON NEXT PAGE


INNER CYLINDER
OUTER CYLINDER
OUTER CYLINDER
INNER CYLINDER
Half CYLINDER (seen from top)
(LIGHT METAL OR MAILING TUBE)
Half O-RING OF APPROPRIATE SIZE
GLUED WITH EPOXY OR SILICONE CEMENT
O-RING
CUT

CONCENTRIC CARDBOARD MAILING TUBES — SPRAYED WITH FLAT BLACK PAINT (MAY BE VARNISHED UNTIL WATERPROOF)

FIG. 2 (CONT.)

SHADOWS

Purpose:

(1) Show geometric relationships between source-object, object-screen distances, and object size vs image size using pinpoint source.

(2) Above using parallel source

(3) Show relationship between image size and image screen angle.

(4) Demonstrate umbra and penumbra due to extended source.

(5) Use lightmeter to analyze "fuzziness" of shadows [*1] from extended (not pinpoint) light sources.

Apparatus:

	EST. COST
Optical Bench (meter stick variety)	$10.00
Holders, assorted	$5.00
Screen (covered with graph paper)	$5.00
Source with reflector & 25 watt bulb	$2.50
Cardboard	----
Hole punch	.70
Tape	----
Compass	.50
*Photometer	$10.00
*1/8 inch lucite rod	$1.00
*Micrometer	$10.00

*Optional

1. See construction details of an inexpensive lightmeter, Jones, E. R. AJP 42(342) 1944, (April Issue).

(1) Background: The geometrical relationships of shadows from point sources may be seen from the following diagram.

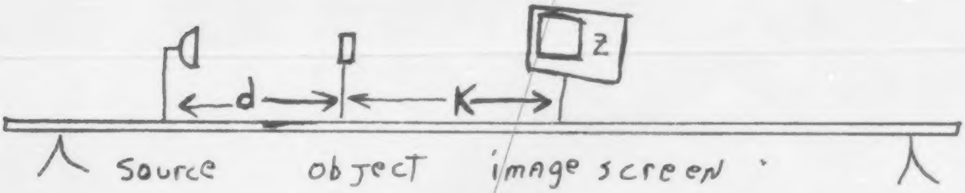


FIG. 1

$$L/d = \frac{Z}{K+d} \Rightarrow Z = \frac{L(K+d)}{d}$$

$$\Rightarrow \boxed{Z = LK/d + L}$$

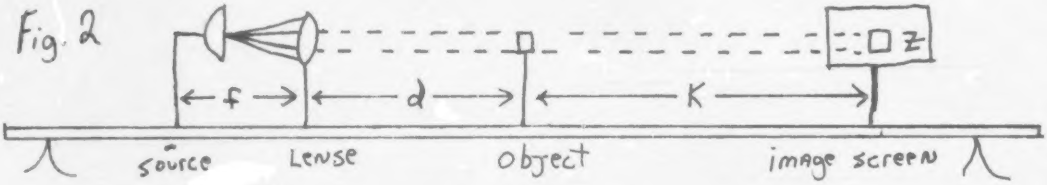
Eq. 1

Procedure: Cut a 3 cm. by 2 cm. rectangular shape from cardboard and place in a holder. Tape a piece of centimeter graph paper to the screen and place in a holder 5 cm. from theobject (rectangle). Next cut a small circular piece of cardboard which will cover the entire front of the light source. Punch a small hole in the center. This hole will serve as an approximate pinpoint light source. Now tape the cardboard to the reflector and place the source about 15 cm. from the objective. Measure the height of the shadow on the screen. Move the screen 2 cm. further away from the object and repeat the measurement. Continue the procedure until the object-screen distance (k in Fig. 1) is at least 3 times the distance between the object and the source (measured from the plane of the cardboard). Notice also how the shadow becomes "fuzzier". This is due to the source being only an approximate point source. Fill in the table below

Object-screen distance k	Z(exp)	Z(cal. from Eq. 1)

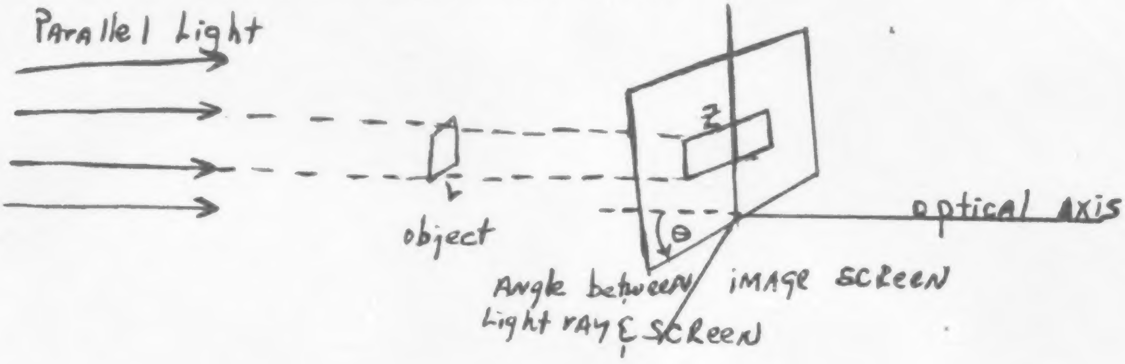
(2) Place short focal length lens in front of point-source so that emitted rays are parallel as shown in Fig. II

Fig. 2



Notice that the size of the shadow produced is almost independent of the distance k by moving the image screen to various positions along the optical bench.

(3) The relationship between the length of the shadow and the angle between the light rays from the parallel source (such as the sun) and the image screen (e.g. the earth). If this angle denoted as Θ in Fig. III is 90^0, the shadow height Z is the same as the object height L; however, if the angle is not 90^0, the shadow appears longer.


Parallel Light
Z
L
object
θ
optical axis
Angle between Light ray & screen
image screen

The general formula is:

$$\frac{Z}{L} = 1/\sin\Theta \qquad \text{Eq. 2}$$

Verify Eq. 2 by making Z measurements at 75^0, 60^0, 45^0, and 15^0. Fill in Table 2. Also, as an extra exercise determine the approximate times of day at which a shadow on earth would bear this same relationship to its object.

Θ	Z	Time of Day
75^0		
60^0		
45^0		
15^0		

(4) Now remove the cardboard cover of the source reflector and punch another hole somewhere at least 2 cm. away from the first. Replace the cardboard and observe the shadow pattern. Now there are two distinct shadows which overlap somewhat in the middle. The darker inner portion is called the umbra while the lighter grey area is called the penumbra. Now remove the cardboard and punch two more holes symmetric about the first two. Replace the cardboard and observe the shadow pattern. Now remove the cardboard altogether and observe the pattern. Describe what you see and explain it in terms of an infinite number of pinpoint sources.

(5) In order to quantify the term "fuzziness" of a shadow a distance called the half-width of the shadow may be defined as follows: first the relative maximum intensity is measured completely outside the shadow. The source without the cardboard is used. Then by carefully measuring the distance required to go from 25% to 75% of the maximum intensity, the half-width is defined as the distance. A graph of this analysis might appear as follows:

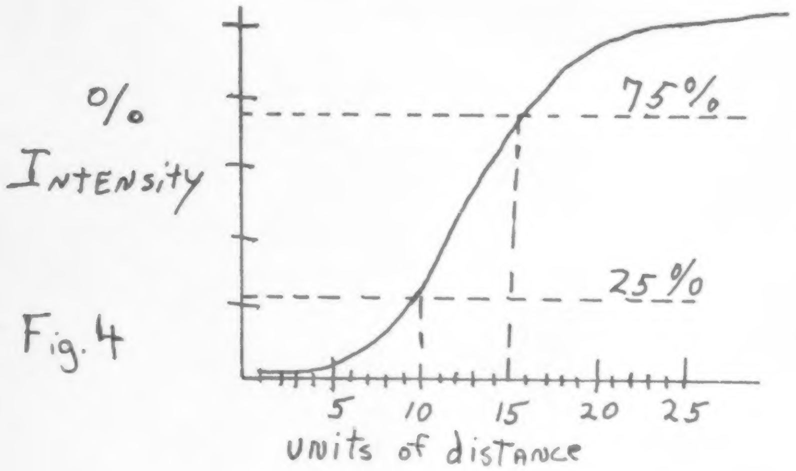
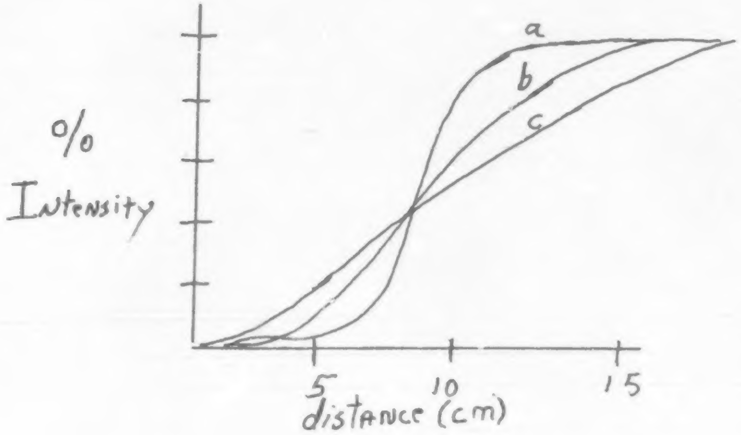


Fig. 4

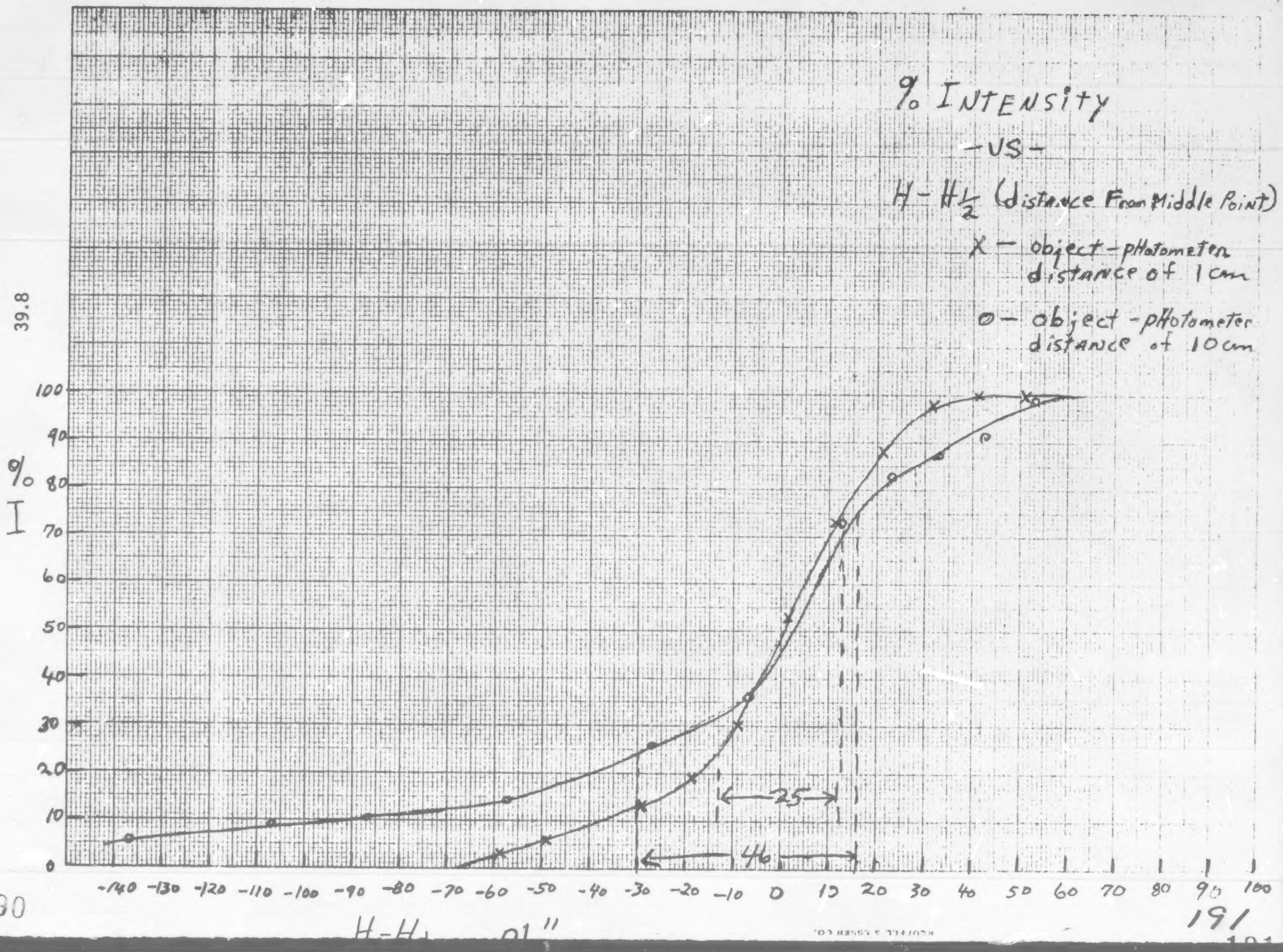
$\Rightarrow \Delta_{\frac{1}{2}} \cong 5$ units

The procedure for measuring this parameter is to replace the screen with a photometer on a lab jack or other suitable device so that its position may be slowly and reproducibly varied. A micrometer or dial indicator is then used to measure the distance traveled. Also, a piece of 1/8 inch lucite makes a good light pipe if properly coated and is much easier to wield attached to a micrometer. A set up is as in Fig. I, except that the screen has been replaced by the photometer. The maximum intensity is recorded and then the apparatus is lowered into the shadow in small increments, recording the relative intensity at each step. The photometer is then moved to a new value of k (distance between object and measuring device) and the procedure repeated. This will yield a set of curves as follows:



*Typical values for two positions are shown in the attached graph. This section is still in very rough shape since I'll probably be the only one who likes it.

39.8



% INTENSITY
-VS-
H - H½ (distance From Middle Point)
X - object-photometer distance of 1 cm
⊙ - object-photometer distance of 10 cm
% I
100
90
80
70
60
50
40
30
20
10
0
-140 -130 -120 -110 -100 -90 -80 -70 -60 -50 -40 -30 -20 -10 0 10 20 30 40 50 60 70 80 90 100
25
46
H-H½ 01''

RESPONSE OF THE EYE AND THE EAR TO INTENSITY CHANGES

Introduction

Both the eye and the ear can respond to a very wide range of intensities; the faintest sounds we can hear are about 10^{-12} times as intense as the loudest we can tolerate without pain, and the faintest light signals we can detect with the unaided eye are about 10^{-12} times as intense as direct sunlight. Because of this wide range of intensities, we express sound intensity, for example, in decibels, which "compresses" the intensity scale logarithmically.

We would like to find a mathematical relation between the intensity of a light or a sound stimulus and the internal sensation produced. However, while it is easy to measure a stimulus in physical terms, it is not so easy to measure a psychological sensation precisely. We are aware that the sun is brighter than a candle, and that a jackhammer is louder than a bumblebee, but evaluations like "twice as loud" or "three times as bright" do not give very precise measurements since they vary somewhat from person to person.

One solution to this problem of psychological measurability is to measure the smallest change in stimulus necessary to produce a change in sensation. If the sensation produced by a stimulus varies as, say, the logarithm of the stimulus (i.e., if the ear works on a decibel-like scale), then the change in stimulus Δs necessary to produce a noticeable change in sensation should be proportional to the stimulus s: $\Delta s = ks$. This relationship was first noticed by Weber in the 19th century and is called Weber's Law. It has been found to be approximately true for all of the physiological sensors with various values for k, the fractional change necessary to be noticed. For instance, for sound intensity changes, $k \approx 0.1$; for smell and taste, $k \approx 0.3$; for skin pressure changes, $k \approx 0.05$.

In this experiment, each student will measure his lab partner's sensitivity to a change in sound and light intensity. When the experiment is completed the tester and the testee (as Flip Wilson would say) change places and repeat the measurements. Afterwards, each lab group will use a light meter and decibel meter to survey light and sound intensities around campus.

In the light portion of this experiment, we will use the fact that light intensity varies inversely as the square of the distance from the source:

$$I = C/D^2$$

In the sound portion we will use the oscilloscope as a voltmeter. The amplitudes on the oscilloscope display are converted to voltages by multiplying by the scale factor used at each point. The intensity of any wave is proportional to the square of its amplitude ,however, we are not interested in absolute magnitudes here, but only in relative changes, so we will use V^2 as intensity.

Apparatus:

A. Light Intensity

1-meter optical bench
photometer head for bench (Bunsen type)
black felt drape
light meter with calibrated filters
2 light sources to mount on bench: 2, 7.5 watt bulbs, 2 100-watt bulbs.

B. Sound Intensity

Audio oscillator
Headphones (monaural or stereo)
Oscilloscope
Intensity switching box (see construction notes below)
Sound pressure level meter (db meter)

Procedure:

A. Light Intensity

We will set $c_1 = 1$ for the 7.5 watt bulbs so $I_1 = c_1/D_1^2$. We must then use one large bulb and one small one to determine c_2 for the 100 watt bulbs.

1. Set up the optical bench with the photometer head at the 50 cm mark, one 100 watt source at 100 cm, and one 7 1/2 watt source on the opposite side of the photometer. Be sure to use the drape to keep stray light from shining in the subject's eyes.

2. Vary the position of the 7 1/2 watt bulb until both halves of the photometer screen appear equally illuminated. Note that the small soruce can be moved back and forth a small distance with no apparent change in the screen. Try to locate the center of this interval. Record the distance

from the 7 1/2 watt source to the photometer head, Calculate $c_2 = c_1 \text{ (D small)}^2/\text{(D large)}^2$.

3. Remove the 100 watt source. Set both 7 1/2 watt sources at 50 cm from the photometer head.
4. Leaving one source fixed, move the other lamp towards or away from the photometer and record the distances at which the subject says that side of the screen is brighter or dimmer.
5. Repeat steps 3 and 4 with the fixed source at 40 cm, 30 cm, 20 cm, and 10 cm, then repeat the entire process with the pair of large sources.
6. Calculate the values in the data chart and graph ΔI vs. I and ΔI/I vs. I.

Data:

small source

D	$1/D^2$	D near	D far	$I=C/D^2$	$\Delta\text{avg.} = \frac{\text{D far}-\text{D near}}{2}$	$\Delta I=c\left(1/D^2-\frac{1}{(D+\Delta)^2}\right)$

Procedure:

B. Sound Intensity

1. Connect signal generator, switch box, headphones; and oscilloscope as shown. Have your instructor check the wiring before proceeding.
2. Turn on oscilloscope and signal generator and set frequency control at 400 Hz. Set signal amplitude at 0.
3. The subject is seated behind the oscilloscope so he cannot see any of the controls and puts on the headphones.
4. Set the oscilloscope vertical sensitivity control at 5 mv/cm and adjust the oscillator amplitude to obtain a wave amplitude of 1 cm.
5. Set the "louder" knob at "max" and the "softer" knob at "min." Throw the switch in either direction and the subject should say "louder" or "softer". Return switch to "normal."

6. Turn both knobs slightly towards "normal" and repeat the process. Be sure to switch from normal to louder and normal to softer irregularly so the subject cannot predict which change will occur next. When you have reached the smallest deviations which the subject can detect, record the oscilloscope amplitudes on the loud and soft settings.

7. Repeat for "normal" voltages of 5, 10, 20, 30, 40, 60, 75, and 100 mv. as shown below.

8. Plot $\Delta s_1/s$ versus s and $\Delta s_2/s$ versus s on linear graph paper.

Data

vert. sens. (mv/cm)	amp. (cm) normal min.max.	V=mv/cm X amp.						$\frac{\Delta s_1}{s}$ =	$\frac{\Delta s_2}{s}$
		V_o	V_{min}	V_{max}	V_o^2	V_{min}^2	V_{max}^2	$\frac{V_{max}^2 - V_o^2}{V_o^2}$	$\frac{V_{min}^2 - V_o^2}{V_o^2}$
5	1	5							
5	2	10							
10	2	20							
10	3	30							
20	2	40							
20	3	60							
50	1.5	75							
50	2	100							

Questions for parts A and B:

1. Do your results verify Weber's Law? Over how many orders of magnitude in I did you test Weber's Law?

2. On the average, were "more intense" or "less intense" changes more noticeable? What would you expect to happen?

3. If you were trying to get better results in this experiment, what changes in procedure would you make?

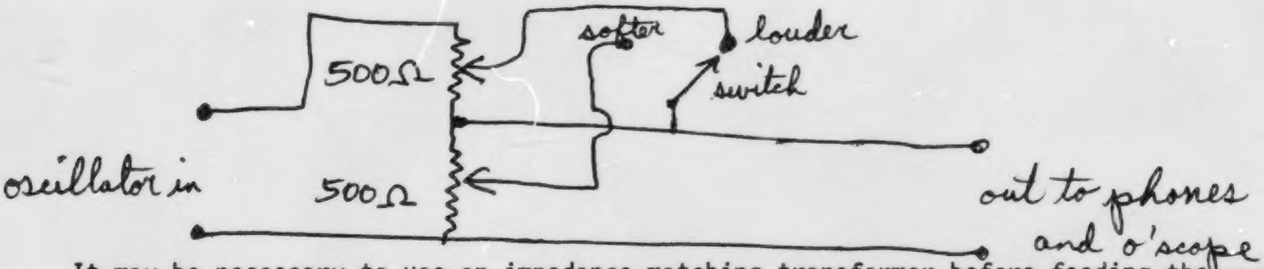
C. Sound and light intensity survey of campus

With a sound pressure level meter (db meter) and a light meter, take readings of the sound and light intensity at various points on campus. You might take light values in reading and study areas, outdoors, in classrooms, etc. Try sound intensities in hallways during class changes, near air conditioners, at a fixed distance from various cars in the parking lot. Are carpeted halls quieter than those with hard floors? Also, in band and concert auditoriums? Compare the exhaust level of different automobiles.

Random notes to the Instructor

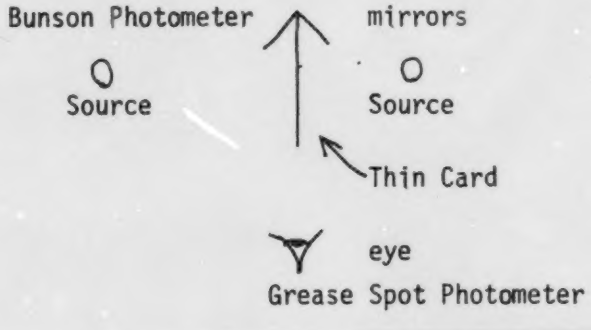
1. This might very well be split into two lab experiments. You could then do either one or both during different lab periods.

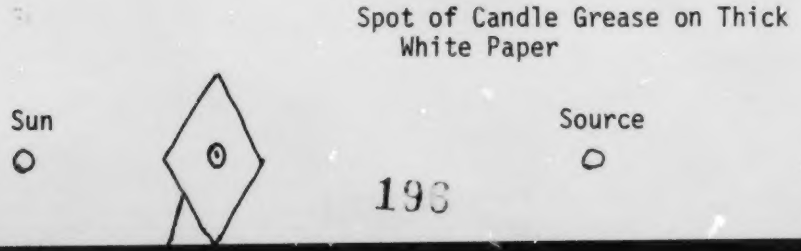
2. The calculation procedure used to get ΔI in the optics part is not exact. However, it introduces little error, and makes calculations simpler than trying to correct for differences in light output of the two supposedly identical lamps.
3. Radio Shack sells a rather decent db meter for about $40.00. Scott sells a much better one for about $100 (Scott Instr.Labs., Cambridge, Mass.).
4. The control box for the audio intensity changes is made of two 500-ohm potentiometers, a stereo headphone jack output, 2 banana plug inputs, and a SPDT center-off switch, preferably spring-loaded to the center position. The schematic is quite simple:


500Ω
500Ω
softer
louder
switch
oscillator in
out to phones
and o'scope

It may be necessary to use an impedance-matching transformer before feeding the signal into the headphones. A 120-volt to 12-volt filament transformer works well.

5. A Bunsen photometer is available from several sources, or you can make your own from a grease spot on a piece of paper and two mirrors, or from two parrafin blocks with a sheet of aluminum foil sandwiched in between.


Bunson Photometer
mirrors
Source
Source
Thin Card
eye
Grease Spot Photometer


Spot of Candle Grease on Thick White Paper
Sun
Source

SCALE CONSTRUCTION

Equipment Pocket Calculator - record player

A. Equitempered Scale

An equitempered scale divides the octave into a fixed number of intervals having the same frequency ratio. The most useful tempered scale in Western music has twelve equal half-steps or semitones of ratio r within the octave. With a fundamental frequency f_o, this gives frequencies of f, rxf, r^2xf, r^3xf, . . . $r^{12}xf$. As the octave is also 2xf, this determines r as $r^{12} = 2$ or $r = \sqrt[12]{2} =$ 1.059463094359295 which is by definition also 100.00 cents.

(1) Assuming that A_4=440 Hz is the standard, calculate the frequencies of C_4 to C_5 i.e., those C's just below and above A_4 and all half steps between.

(2) Find the frequency of A_o and C_8, the lowest and highest notes on the piano.

B. Pythagorean Scale

The Pythagorean scale is defined by the following sequence beginning with the fundamental frequency f say for C_4

a) 2f = octave of C_5

b) C_5 down a fifth or $\frac{2}{3} \times 2f = \frac{4}{3}f = F_4$

c) C_4 up a fifth or $\frac{3}{2} \times f = \frac{3}{2}f = G_4$

d) G_4 up a fifth & down an octave or $\frac{3}{2} \times \frac{1}{2} \times \frac{3}{2}f = \frac{9}{8}f - D_4$

e) D_4 up a fifth or $\frac{3}{2} \times \frac{9}{8}f = \frac{27}{16}f = A$

This gives the pentatonic scale (mode)

f) A_4 up a fifth down an octave $= \frac{3}{2} \times \frac{1}{2} \times \frac{27}{16}f = \frac{81}{64}f = E_4$

g) E_4 up a fifth $= \frac{3}{2} \times \frac{81}{64}f = \frac{243}{128}f = B_4$

(1) Compute the frequencies of each note, taking f = C_4 to be determined by the equitempered scale.

(2) By looking up the cents equivalent to each interval, list the discrepency of each note from the equitempered note.

Just Scale

A major triad is a set of three notes with frequencies which are in the ratio of 4:5:6 or f, $\frac{5}{4}f$, $\frac{3}{2}f$, e.g. CEG. A scale can be built from a frequency f

(a) by using it as the bottom note of a major triad giving CEG with the frequencies f, $\frac{5}{4}f$, $\frac{3}{2}f$.

(b) by using the G as the bottom note of a new triad giving GBD as $\frac{3}{2}$f, $\frac{15}{18}$f, $\frac{9}{4}$

(c) by using the C as the top note of a triad to get FAC as $\frac{2}{3}$f, $\frac{5}{6}$f, f

(d) Those notes lying outside the octave are moved by an octave (F and A up one octave) D down one octave to get C1, D$\frac{9}{8}$, E$\frac{5}{4}$, F$\frac{4}{3}$, G$\frac{3}{2}$, A$\frac{5}{3}$, B$\frac{15}{8}$, C2

(1) Repeat parts B1 and B2 for the Just scale

Comparison

1. Compare the scales by listening to recorded comparisons. Verify that the most noticeable differences in cents are the most noticeable aurally. Which scale do you find preferable and why?

Equitmepered Divisions of the Octave

Equipment Pocket Calculator

Description:

The octave can be broken into divisions other than the standard twelve divisions. If one breaks the octave into N parts then the equitempered frequencies must be f, f x D, f x D^2, fD^3, fD^4, . . . fD^N where the frequency of f is fundamental and f x D^N is one octave higher. Thus $D^N = 2$ or $D = N\sqrt{2}$. These members and their equivalent in terms of cents can be obtained from the instructor for all N from 1 to 100. The higher frequencies D^2, D^3, etc. must be obtained by successive multiplication. However the cents equivalents may be obtained by multiplying the cents by m for the mth note. (See page 130 Backus).

Procedure:

(1) Choose some division of the octave other than 12. Compute the cents equivalent for each note (relative to the fundamental).

(2) Compare the available notes to the notes on the equitempered scale (cents = 0, 100, 200, 300, . . . 1200) to see if they could be discriminated by the ear.

(3) Compare the intervals also to the just intervals to see if there would be a perceptible audible difference. (The frequencies and cent equivalents for these are available from the instructor).

Division of the Octave into N intervals. The semitone frequency = $\sqrt[N]{2}$ is computed and also expressed in cents.

N	$\sqrt[N]{2}$ = FREQUENCY	CENTS
1	2	1200
2	1.414213562373095	600
3	1.259921049894873	400
4	1.189207115002721	300
5	1.148698354997035	240
6	1.122462048309373	200
7	1.104089513673812	171.43
8	1.090507732665257	150
9	1.080059738892306	133.34
10	1.071773462536293	120
11	1.065041089439962	109.1
12	1.059463094359295	100
13	1.054766076481647	92.31
14	1.050756638653219	85.72
15	1.047294122820627	80
16	1.044273782427414	75
17	1.041616010650584	70.59
18	1.039259226031843	66.67
19	1.037155044446192	63.16
20	1.035264923841377	60
21	1.033557783007028	57.15
22	1.03200827973421	54.55
23	1.030595544752009	52.18
24	1.029302236643492	50
25	1.028113826656066	48
26	1.027018050708772	46.16
27	1.026004484707038	44.45
28	1.025064211965874	42.86
29	1.024189560248997	41.38
30	1.023373891996775	40
31	1.022611435601268	38.71
32	1.021897148654117	37.5
33	1.021226606315364	36.37
34	1.020595909579586	35.3
35	1.020001609421199	34.29
36	1.019440643702145	33.34
37	1.018910284405251	32.44
38	1.018408093274102	31.58
39	1.017931884337391	30.77
40	1.017479692102686	30
41	1.017049744443627	29.27
42	1.016640439391935	28.58
43	1.016250325193552	27.91
44	1.015878083105551	27.28
45	1.015522512504275	26.67
46	1.015182517950348	26.09
47	1.01485709791702	25.54
48	1.014545334937524	25
49	1.014246386967327	24.49
50	1.013959479790029	24

```
 51     1.013683900322685      23.53
 52     1.0134189906987       23.08
 53     1.013164143024915      22.65
 54     1.012918794724947      22.23
 55     1.012682424393701      21.82
 56     1.012454548098765      21.43
 57     1.012234716073474      21.06
 58     1.012022509754105      20.69
 59     1.011817539120121      20.34
 60     1.011619440301922      20
 61     1.01142787342522      19.68
 62     1.01124252066518      19.36
 63     1.011063084486896      19.05
 64     1.0108892860517       18.75
 65     1.010720863771376      18.47
 66     1.010557571994473      18.19
 67     1.010399179810882      17.92
 68     1.010245469962418      17.65
 69     1.010096237848608      17.4
 70     1.009951290618116      17.15
 71     1.009810446337321      16.91
 72     1.009673533228511      16.67
 73     1.009540388970987      16.44
 74     1.009410860059099      16.22
 75     1.009284801211874      16
 76     1.009162074829461      15.79
 77     1.009042550492123      15.59
 78     1.008926104497941      15.39
 79     1.008812619435776      15.19
 80     1.008701983790399      15
 81     1.008594091576999      14.82
 82     1.008488842002541      14.64
 83     1.008386139151709      14.46
 84     1.008285891695374      14.29
 85     1.008188012619719      14.12
 86     1.008092418974348      13.96
 87     1.007999031637824      13.8
 88     1.007907775099265      13.64
 89     1.007818577254717      13.49
 90     1.007731369217151      13.34
 91     1.007646085139043      13.19
 92     1.007562662046559      13.05
 93     1.007481039684486      12.91
 94     1.007401160371091      12.77
 95     1.007322968862183      12.64
 96     1.007246412223704      12.5
 97     1.007171439712219      12.38
 98     1.007098002662763      12.25
 99     1.007026054383499      12.13
100      1.006955550056719      12
```

42.5

Fundamental frequency ratios in music are defined, computed, and given in equivalent cents.

```
              1:1                  UNISON
0   CENTS OR FREQ OF   1
              2*(1:1200):1        ONE CENT
1   CENTS OR FREQ OF   1.000577789506555
              1.0025:1             BEST PITCH DISCRIMINATION(≥ 1000Hz)
4.33   CENTS OR FREQ OF   1.0025
              81:80                SYNTONIC COMMA
21.51   CENTS OR FREQ OF   1.0125
              531441:524288        PYTHAGOREAN COMMA
23.47   CENTS OR FREQ OF   1.013643264770508
              256:243              PYTHAGOREAN DIATONIC SEMITONE
90.23   CENTS OR FREQ OF   1.053497942386831
              2*(1:12):1           EQUITEMPERED SEMITONE
100   CENTS OR FREQ OF   1.059463094359295
              16:15                JUST DIATONIC SEMITONE
111.74   CENTS OR FREQ OF   1.066666666666667
              2187:2048            PYTHAGOREAN SEMITONE
113.69   CENTS OR FREQ OF   1.06787109375
              1.1:1                WORST PITCH DISCRIMINATION(30 Hz)
165.01   CENTS OR FREQ OF   1.1
              10:9                 JUST WHOLE TONE
182.41   CENTS OR FREQ OF   1.111111111111111
              2*(2:12):1           EQUITEMPERED WHOLE TONE
200   CENTS OR FREQ OF   1.122462048309373
              9:8                  PYTHAGOREAN WHOLE TONE
203.92   CENTS OR FREQ OF   1.125
              32:27                PYTHAGOREAN MINOR THIRD
294.14   CENTS OR FREQ OF   1.185185185185185
              2*(3:12):1           EQUITEMPERED MINOR THIRD
300   CENTS OR FREQ OF   1.189207115002721
              6:5                  JUST MINOR THIRD
315.65   CENTS OR FREQ OF   1.2
              5:4                  JUST MAJOR THIRD
386.32   CENTS OR FREQ OF   1.25
              2*(4:12):1           EQUITEMPERED MAJOR THIRD
400   CENTS OR FREQ OF   1.259921049894873
              81:64                PYTHAGOREAN MAJOR THIRD
407.83   CENTS OR FREQ OF   1.265625
              4:3                  PYTHAGOREAN•JUST FOURTH
498.05   CENTS OR FREQ OF   1.333333333333333
              2*(5:12):1           EQUITEMPERED FOURTH
500   CENTS OR FREQ OF   1.334839854170034
              2*(6:12):1           EQUITEMPERED DIMISHED FIFTH
600   CENTS OR FREQ OF   1.414213562373095
              2*(7:12):1           EQUITEMPERED FIFTH
700   CENTS OR FREQ OF   1.498307076876681
              3:2                  PYTHAGOREAN JUST FIFTH
701.96   CENTS OR FREQ OF   1.5
              128:81               PYTHAGOREAN MINOR SIXTH
792.18   CENTS OR FREQ OF   1.580246913580247
              2*(8:12):1           EQUITEMPERED MINOR SIXTH
800   CENTS OR FREQ OF   1.587401051968199
              8:5                  JUST MINOR SIXTH
813.69   CENTS OR FREQ OF   1.6
```

```
            5÷3                JUST MAJOR SIXTH
 884.36  CENTS OR FREQ OF   1.666666666666667
            2*(9÷12)÷1         EQUITEMPERED SIXTH
 900  CENTS OR FREQ OF  1.681792830507429
            27÷16               PYTHAGOREAN MAJOR SIXTH
 905.87  CENTS OR FREQ OF  1.6875
            2*(10÷12)÷1         EQUITEMPERED DIMISHED SEVENTH
1000  CENTS OR FREQ OF  1.781797436280679
            15÷8               JUST SEVENTH
1088.27  CENTS OR FREQ OF  1.875
            2*(11÷12)÷1        EQUITEMPERED SEVENTH
1100  CENTS OR FREQ OF  1.887748625363387
            243÷128             PYTHAGOREAN SEVENTH
1109.78  CENTS OR FREQ OF  1.8984375
            2÷1                OCTAVE
1200  CENTS OR FREQ OF  2
```

SIMPLE HARMONIC MOTION AND RESONANCE

Introduction:

We have all seen examples of repetitive motion; waves moving toward a beach, the swaying of trees in the wind and many others. Even more we have heard the results of such motion, such as sound waves from a vibrating violin string or from a vibrating drum. One of the most familiar types of motion which repeats itself is called simple harmonic motion (SHM). The simple pendulum and a mass on the end of a spring are examples of SHM. An object executing SHM has a natural frequency. If a force is applied to the object in motion, it can be made to vibrate at the frequency of the applied force. When the force frequency is the same as the natural frequency, the amplitude of the resultant motion can become quite large. This phenomenon is called resonance.

Objective:

The objective of this experiment is to familiarize you with some of the characteristics of SHM and resonance.

Theory:

Simple harmonic motion is defined as any motion which results when the total force, F, on an object is (1) proportional to the objects distance, X, from its equilibrium position and (2) directed toward the equilibrium position. This relationship can be expressed as

$$F = -kx \qquad (1)$$

where k is the constant of proportionality between the force and the displacement distance.

When the SHM is due to the force exerted on a mass by a stretched spring, k is called the spring constant. The distance from the equilibrium position of a mass executing SHM is a sinusoidal function of time. It can be thought of as the motion of the shadow of an object moving in a circle at constant speed (Figure 1)

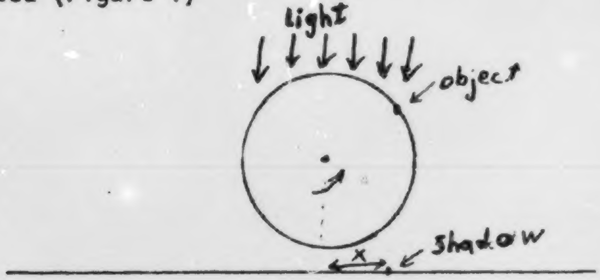


Figure 1

Unless a spring is stretched so far that Eqn. 1 is no longer true, the time for a mass to complete one cycle of its motion is independent of the distance the mass moves. The time for completing one cycle is a period of the motion and is designated by T. The period depends only on the mass, m, of the moving object and the spring constant;

$$T = 2\pi \sqrt{m/k} \tag{2}$$

The number of cycles completed by the mass in one second is the frequency, F. The period and the frequency are reciprocal, i.e.,

$$F = \frac{1}{T} \tag{3}$$

The maximum distance the mass moves from its equilibrium position is its amplitude, A. If a mass is displaced from its equilibrium position by a given distance and released, it will execute SHM with an amplitude equal to its original displacement. This is the ideal case. In reality there are always frictional forces present which cause the amplitude to decrease as time passes.

Thus far only SHM with no driving force has been discussed. Consider a mass which has two forces, one due to stretched springs and one sinusoidal, acting on it (Figure 2)

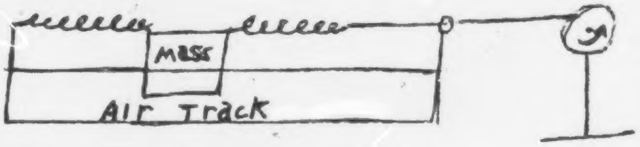


Figure 2

If the frequency of the driving (sinusoidal) force is not near the natural frequency of the mass attached to the springs, the amplitude of the resulting motion will be small. However, as the driving frequency approaches the natural frequency the amplitude increases rapidly. When the two frequencies are equal the amplitude is a maximum and resonance has been reached.

Apparatus:

- Stopwatch or Timer
- Air Track
- Glider
- Two Springs
- Variable Speed Motor With Sinusoidal Drive
- Spark Source
- Set of Weights
- Balance
- 4' Magnetic Recording Tape
- Spark Tape
- Roll 10" White Paper
- String

Procedure:

1. Measure and record the mass of the glider.
2. Determine the spring constant for each spring. This can be done using the arrangement in Figure 3 and recording the elongation of the spring as a function of the force stretching it.

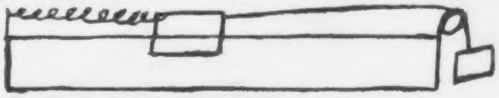


Figure 3

Make a graph of force on the spring versus its elongation. The spring constant is the slope of this graph.

3. Using the arrangement in Figure 4, stretch each spring about 20 cm. Displace the glider about 5 cm. from its equilibrium position and time at least 10 cycles of the motion. Calculate the period. Repeat using both larger and smaller amplitudes. Is their any significant change in the period?

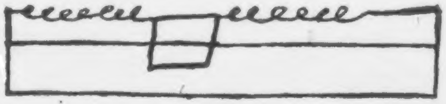


Figure 4

4. Compare the measured period with the period calculated using Eqn. 2. (You are using two springs, so what is the spring constant for the motion?)

5. Set up the SHM with the sinusoidal driving force as shown in Figure 2. Use a small amplitude, .5-1.0 cm., for the driving force. Adjust the frequency of the driving force until resonance is reached.
6. Increase the frequency of the driving force to at least twice the resonant frequency. Lower the frequency in steps, recording the amplitude at each frequency. This recording may be done with the spark timer. Tape a piece of spark recording paper parallel to the track on the top of the support beam below the track. Center the spark paper on the equilibrium position of the glider. When you wish to record the amplitude simply put a piece of white paper over the spark recording paper and lower the bottom spark wire until it is just above the spark paper. If you pull the white paper through at a constant speed you will see that the motion is sinusoidal and from the number and position of spark holes you can measure the amplitude and the frequency.
7. Plot a graph of amplitude verses frequency and compare the resonant frequency with the one measured in part 3.

Helpful Hints:

1. You can make a sinusoidal drive from the cenco centripetal force apparatus. Take off the enclosure with the mass and spring and put a small nail in the vacant hole. Loop the string (the other end of which is attached to a spring) around the nail. The slight off-center position is enough displacement to drive the system. *
2. Try to find two identical springs which will give you a resonant frequency of at least 2 Hz. The Cenco apparatus does not operate well under about 1 Hz.
3. If the driving amplitude is too large you may get some nonlinear effects, such as jump amplitudes, described in several of the references. Some good students might want to study these.
4. A better spark recorder is described in reference 7.
5. If you wish you can obtain a sinusoidal force drive from the Ealing Corporation.

Student Glider Sine Drive	#CB33-0357	$30.00
Universal Gear Motor	#CB34-3152	$49.50
Controller for Gear Motor	#CB34-3160	$52.50

* The frequency of the motion can also be measured using the revolution counter on the apparatus.

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PITCH

There are three primary sensations which describe a musical sound: pitch, loudness, and timbre. Pitch is frequently described as the sensation of "altitude" or "height." The psychologist might define pitch as "that attribute of auditory experience which determines the positions of sounds in a psychological continuum from the lower to the upper frequency limit of tonal sensation." To the musician, pitch relates a sound to some note on a musical scale.

Pitch is determined mainly by frequency. However, there are secondary effects such as loudness and timbre which might be expected to influence our sense of pitch; some of these will be studied in Part A of this experiment. Part B, which introduces some complex sounds whose pitch is not a simple function of frequency, will be of interest to the more serious student of musical acoustics.

Part A: Pitch of Musical Tones

In this part of the experiment, the pitch of pure tones and those with harmonic overtones, such as musical tones, will be studied. Three questions will be investigated: does pitch change with loudness? does pitch change with timbre? how does vibrato effect pitch?

These questions were intensively studied about 40 years ago by Harvey Fletcher at the Bell Telephone Laboratories, by S. S. Stevens at Harvard, by Carl & Harold Seashore at the University of Iowa, and by others. The results of these experiments were in good agreement with the classical "place" theory of pitch perception developed by Ohm,von Helmholtz and von Békésy. Attention is called to a paper by Dr. Fletcher in the October 1934 issue of The Journal of the Acoustical Society of America as well as books by von Helmholtz, Seashore, and von Bekesy.

1. Does pitch change with loudness?
    a) Using a reference tone of variable frequency at 40db to determine apparent pitch, listen to tones from 50 Hz to 10,000 Hz at levels of 60, 80 and 100 db. Determine the apparent pitch of each tone by adjusting the frequency of the reference tone and switching back and forth. (If a frequency counter is available, the exact frequency of each tone can be determined with precision; otherwise the change

in frequency can be obtained from the oscillator dial even though the actual frequency is slightly different from the indication on the dial).

Make a graph of pitch deviation vs frequency (on semi-log graph paper) with three curves to represent the three intensity levels. Use your own judgment to determine how many data points are necessary to obtain reasonably smooth curves. Interpret your results.

b) Repeat the experiment of Part A using a tone rich in harmonics such as a square wave or a sawtooth wave in place of the pure tone. Interpret your results.

2. Does pitch change with timbre?

Two methods will be outlined for making this experiment, the choice depending on equipment available.

a) Set a function generator to a given frequency, say 440 Hz, and listen to all available waveforms (e.g. sine, square, sawtooth, pulse, etc.). If any difference in apparent pitch is noted, arrange them in order from highest to lowest. Repeat the experiment at 4 other frequencies, perhaps 2 octaves up and 2 octaves down from 440 Hz. Discuss your results.

b) Using musical instruments or tape recordings of musical instruments, measure the exact frequency with a frequency counter, and also determine the frequency of the pure tone (at the same loudness level) which has the same apparent pitch. Discuss your results.

3. What is the apparent pitch of a tone with frequency modulation (i.e., a type of vibrato)?

For a source of frequency-modulated sound, use a dual function generator, a musical synthesizer, or a voltage-controlled function generator together with another oscillator capable of generating a low-frequency tone.

a) With a fixed modulation frequency of 7 Hz (an "average" rate for vibrato used by both vocalists and instrumentalists), vary the amount of frequency change (i.e., the "depth" of the vibrato) from a full tone (±6%) down to the smallest amount perceptible. Begin with a center frequency of 440 Hz, but try several others as well. Determine the "average" pitch by comparison with a pure tone of the same loudness, and also try to estimate the depth of vibrato if you have a "musical" ear which recognizes small intervals.

Measuring the actual frequency change Δf may be fairly difficult, and your instructor will help you devise a method which makes best use of the equipment available.

b) In this part of the experiment, the "depth" of modulation will be kept fixed as the modulation rate is varied. Begin with a center frequency of 440 Hz and a frequency swing of 1/4 tone (±1.5%). Determine the apparent pitch and depth of vibrato (if you can) for modulation frequencies of 3, 4, 5, 6, 7, 8 and 9 Hz. Repeat for frequency swings of a semi-tone (±3%) and 1/8 of a whole tone (±0.7%). Discuss your results.

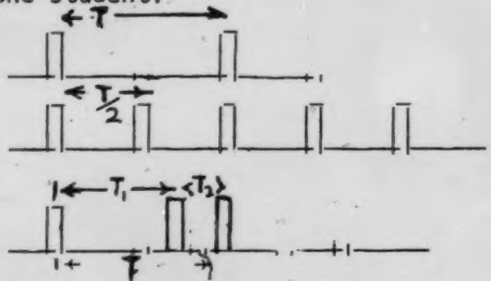
c) What combination of depth and rate of vibrato did you find most pleasing? How does that compare to that used by most performers of music?

Part B. Pitch of Complex Sounds

Many of the sounds that we hear do not consist of a fundamental note plus harmonic overtones; yet our ears are able to assign a pitch to these sounds. The manner in which this is done has been the subject of much research recently, but is not completely understandable. A good review of historical and current understanding of pitch perception is contained in a recent article by Wightman and Green (American Scientist 62, 208 (1974).

In this part of the experiment, 4 different sounds are presented. These may be generated in the laboratory or they may be pre-recorded on tape. If a spectrum analyzer is available, the sound spectrum should be recorded, since it is important to understanding the pitch perception. If no analyzer is available, the sound spectrum will be furnished the student.

1. The first set of sounds to be studied consists of a set of three pulse trains used by Seebeck in 1841 and described in the paper by Wightman and Green. In each case, the pitch is determined by comparison to a pure tone as in Part A.



One way to generate these pulse patterns (in fact the method used by Seebeck) is with a disc which rotates in front of an air jet in the manner of a siren. The pattern of the holes will determine the pattern of pulses.

Another method is to do it electronically with "flip-flop" circuits. Yet another method would employ a small digital computer plus a digital-to-analogue converter.

2. When tones with frequencies of 600, 800, 1000, and 1200 Hz are sounded together, the pitch that is heard is usually identified as 200 Hz, because the frequencies presented to the ear are harmonics of a "missing" 200 Hz fundamental. This may be illustrated by recording or listening to the sound of a musical instrument after the signal has gone through a high-pass filter that surpresses the fundamental and possibly the second harmonic as well. (This is not unlike what happens when music is heard on a transister radio with a tiny loudspeaker that has little or no ability to reproduce tones of low frequency, yet bass notes are heard).

   Listen to the sound of a musical instrument as one after another of the lower harmonics is deleted. Does the pitch change?

3. Bells, gongs and tympani have many overtones which are not harmonics of the fundamental. Identify the pitch of these percussive sounds, and see if you can relate the pitch you hear to the sound spectra.

4. A very interesting experiment on pitch, first performed by Bilsen and Ritsma, is described in a recent article by Biken and Goldstein (J. Acoustical Society of America 55, 292 (1974)). White noise is delayed from 1 to 10 milliseconds and mixed with the same undelayed white noise. The ear hears a rather faint pitch at a frequency 1/T, where T is the delay time. This is termed "monochotic repetition pitch" (MRP). Another version of this experiment (termed "dichotic repitition pitch" or DRP) supplies delayed white noise to the other ear. Again a pitch of frequency 1/T is heard, but somewhat fainter.

   Either MRP or DRP can be generated by using a tape recorder with separate record and playback heads to delay the signal. If the spacing between the heads is fixed, then one delay time can be obtained for each tape speed. The delay time is easily calculated.

Part C. Interpretation of These Experiments

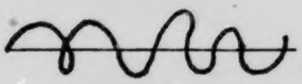
The student who wishes to pursue this subject further might wish to examine these experiments, especially those of Part B, in light of different theories of pitch. Is the classical place theory incorrect? Can it be modified to agree with these results? Does the residue theory of Shouten and others have validity?

What are the "pattern-transformation model" of Wightman (J. Acoustical Society Am. 54, 507 (1973)) and the "optimum processor theory" of Goldstein (ibid 54, 1496 (1973))? What significance do the unusual sounds of Part B have for the musician?

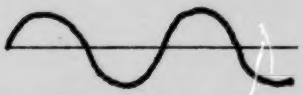
POLARIZATION

I. Introduction

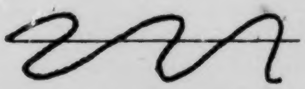
The effects of "polarization" were observed as early as 1669 by the Danish scientist Erasmus Bartholimus who noticed that crystals of Iceland spar (calcite) had the curious property of splitting a ray of light into two rays. The explanation of this property was proposed by Young and Fresnel around 1820. This explanation was based upon the idea of the wave nature of light. Before this time, those scientists who believed light was a wave generally assumed that, like sound waves, it was a longitudinal wave. Young and Fresnel showed that if light were a transverse wave phenomenon, it would account for polarization. In a transverse wave, the wave motion is across and at right angles to the direction the wave is traveling. Thus the motion of the wave disturbance is not always in the same direction, but it is always perpendicular to the direction of propagation. If a beam of light has its wavelike oscillations in predominantly one direction rather than in random directions, it is said to be polarized. Thus a polarized wave is the simplest kind of transverse wave. An unpolarized transverse wave is a more complicated thing since it is a mixture of various transverse motions.



unpolarized wave
on a rope

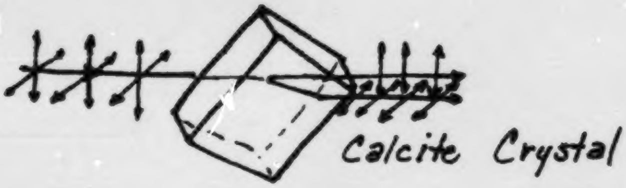


vertically polarized
wave on a rope



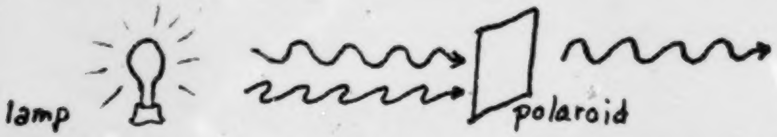
horizontally polarized
wave on a rope

The light which is incident on a calcite crystal is unpolarized. The properties of the calcite are such that is separates the unpolarized light into two polarized components and transmits them through the crystal in different directions and with different speeds as shown below.



Although the property of polarization was discovered and then explained in the nineteenth century, practical applications were not feasible, because polarizing substance were scarce and fragile. One of the best was "herapathite" or sulfate of iodo-quinine, a synthetic crystalline material. The crystals themselves were so fragile that there seemed to be no way of using them. In 1928 Edwin H. Land, while still a freshman in college, invented a polarizing plastic sheet he called "Polaroid". His first polarizer consisted of a plastic film in which many microscopic crystals of herapathite were imbedded. When the plastic is stretched, the needle-like crystals line up in one direction so that they all act on incoming light in the same way.

When one places a sheet of this Polaroid material in a beam of light, it will filter out all light with oscillations in other than one preferred direction as shown below-



If another sheet of Polaroid is placed in the beam of light and rotated one finds that for a certain orientation, all the light is eliminated. Two sheets of Polaroid arranged in this manner at right-angles are said to be "crossed."

Quite a number of transparent substances placed between crossed Polaroids cause the light to be transmitted again by the second Polaroid. Many such substances are optically active, for example, quartz and sugar solutions, and when viewed between crossed Polaroids, they often produce strongly colored patterns. Sheets of cellophane and mica, silk, flax, cotton, white hair, and wool fibers frequently show optical properties of this kind and yield beautiful and varied color effects on rotation between crossed Polaroids.

II. Objective:

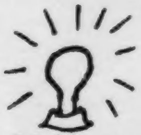
In this experiment you will observe the various effects discussed above and make some measurements to determine the degree of polarization for various configurations of the Polaroids.

III. Equipment:

Light source, Polaroid sheets, detector to measure light intensity, celophane, tape, protractor.

IV. Procedure:

1. Arrange your experimental apparatus as shown below-


light source

Polaroid

Detector

What happens to the intensity of the light when the Polaroid sheet is removed?

2. Now place a second Polaroid sheet in the beam and rotate it until the maximum amount of light is reaching the detector. Record the intensity of the light when the Polaroids are in this position. (The angle between the two is 0^o). Now rotate the second Polaroid through various angles from 0^o to 180^o. Observe and record the intensity of the light for about 7 or 10 angles.

3. Arrange the Polaroids so that they are crossed, that is so that a minimum amount of light is reaching the detector. Place a piece of cellophane between the two Polaroids. Observe what happens to the intensity of the light. Rotate the piece of cellophane and note the results.

4. Use the cellophane and cellophane tape to form a design with varying thicknesses of these materials. Place a sheet of Polaroid material behind your creation and another in front of it. What effects do you observe as you rotate the polaroid in front? In back?

V. Other Questions

1. Using your data from step 2 above plot a graph of $\cos^2\theta$ versus the intensity.

2. For what angle does the maximum intensity occur? The minimum? Explain this in terms of what you have learned about the process of polarization.

3. What happens when the cellophane is inserted? Can you explain why this would happen?

4. Why are polaroid lenses used in sunglasses? Explain how they protect your eyes from sunlight.

5. One way to achieve privacy in apartments facing each other across a narrow courtyard while still allowing residents to enjoy the view of the courtyard and the sky above the courtyard is to use polarizing sheets placed over the windows. Explain how the sheets must be oriented for maximum effectiveness.

6. To prevent car drivers from being blinded by the lights of approaching automobiles polarizing sheets could be placed over the headlights and windshields of every car. Explain why these sheets would have to be oriented the same way on every vehicle and must have their polarizing axis at 45° to the vertical.

Notes To The Instructor:

1. Light Source: clear 150 watt globe mounted in a housing with an approximately 2 inch diameter aperture is a suffuciently intense source; another source is to use a slide projector for the beam of light.

2. A photographic light meter is sufficiently sensitive for the detector although it proved somewhat difficult to use. An inexpensive photocell would be a better detector if a voltmeter, ammeter, or oscilloscope suited to the photocells output is available.

3. The light source, polarizing sheets and detector should be mounted in some way so that their relative positions could be held constant during the experiment.

4. Avoid placing the Polaroid material very close to the light source to avoid damaging it due to heat.

Equipment:

Polaroid Sheets 6" x 6" pkg. of 4 Stock #60637 Edmund Scientific Co. Barrington, N.J. 08007	- $2.50
Photocell: International Rectifier Silicon S4M-C High efficiency photovoltaic output up to 0.4 volts up to 40 mA	- $4.40

References:

"Polarized Light"

Selected Reprints: American Association of Physics Teachers

Resource Letter PL-1

address: 335 East 45th. Street
New York, N. Y. 10017

Price: $2.00 each

STEREOSCOPIC PICTURES

Purpose:

To make and study stereo photographs

Description:

Because the eyes are separated by a small distance horizontally, images perceived by each eye arrive from slightly different angles. This difference produces depth perception or stereopsis. The fact that the brain sees two slightly different pictures and fuses them into a three dimensional image (this process is known as fusion) can be used to produce stereo or three dimensional photographs. These stereo pictures are much used in aerial and satellite reconnaisance work.

Stereo photographs are produced by taking pictures with two different cameras set at a fixed distance apart. The same effect can be produced with one camera. A picture is taken; the camera is moved some distinct distance horizontally; and, then, a second picture is taken.

If available, the student should examine a set of aerial stereo photographs and, perhaps, a set of stereo molecular model drawings with a stereo viewer. With practice some students may find that they can observe the stereo effect without the aid of the viewer. The student should notice an exaggerated depth in the stereo aerial photographs.

After examining the available photographs and figures, the student should proceed to create his own stereo photographs. A simple Polaroid camera can be used to do this. The object to be photographed and viewed should have quite discernable and measurable depth (figures made from Tinkertoys are excellent for this purpose). A photographic meterstick should be placed directly in front of the figure to be photographed so that a scale factor can be established for the finished photos. The camera is to be placed approximately one and one half meters from the objects to be photographed. This distance should be roughly at least ten times the depth of the figure being photographed. The student should record the distance from the figure to the lens of the camera. After the initial picture is taken, the camera should be displaced horizontally to take the second picture. The amount of this displacement should be recorded (a displacement of ten centimeters proves to work quite well). By imaging the resulting photos beneath the stereo viewer, the student should be able to observe a three dimensional picture.

To observe the effect of the amount of separation between camera positions upon the stereoscopic image, a third photo with the camera displaced even further, needs to be taken. In this third picture, it is recommended that the student should double the initial displacement of the camera. If the initial displacement was ten centimeters, in doubling this, the final camera position will be twenty centimeters from the initial camera position. Now one can compare the stereo effect with a camera being displaced two different distances.

If one knows the distance from the camera to the source, the displacement between camera positions, and the scale factor of the resulting photographs, it is possible to make actual depth calculations of objects.

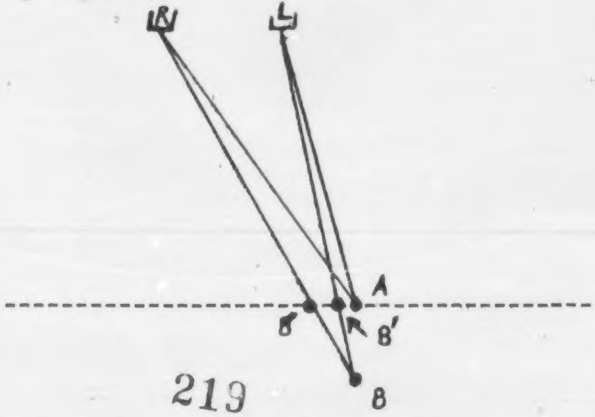
An approximate formula to calculate depth or height of an object is given by the following:

$$\text{height} = \frac{\text{image difference (scaled)}}{a}$$

$$\text{where } a = \frac{\text{camera separation}}{\text{distance from camera to object}}$$

The scaled image difference is the only concept in the above formula which needs explanation. It is calculated from the pair of photos which constitute the stereo picture. Assume one wants to find the depth from one point to another. Measure the distances between the two points on the individual photographs with a comparator. The difference in these two distances is the image difference. The image difference needs to be multiplied by the scale factor in order to have the scaled image difference. The scale factor can easily be found from the meterstick in the photos. The depth calculated by means of the above formula can be compared with experimentally measure depths of the real object.

The formula for calculating heights in the stereo photos can be derived from the following figure:



L and R indicate cameras separated by some fixed distance. One wishes to find the distance between the two points A and B. Point B appears to lie at B" according to the right camera. Point B appears to lie at B' according to the left camera. If the distance from the cameras to the object is large compared to the depth AB and if the angle LRB is approximately a right angle, one has a situation of similar triangles. The ratios of these similar triangles give the aforementioned formulas.

QUESTIONS

1. What creates the exaggerated depth of aerial stereo photographs?
2. What is the effect of increasing the camera separation in viewing stereo photographs?
3. Is there a noticeable difference in the resulting image if the positions of theindividual photographs are interchanged? (i.e.), is it necessary that the picture taken by the camera in the right hand position be viewed by the right eye and the picture taken in the left hand position be viewed by the left eye; or can they be interchanged with no effect?

Technical Note:

Some students may be able to fuse the pictures without the aid of the stereo viewer. Other students may never be able to fuse the images without the aid of a stereo viewer. Inability to fuse the images by a student is generally a physical eye defect known as strabismus.

Equipment:

Stereo viewer (Edmund Scientific - $17.00)
Comparator (Edmund Scientific - $24.00)
Polaroid camera ($30.00) (Colorpack model)

References

Wilman, C. S., Seeing and Perceiving, London: Pergamon Press, 1966.

Hiles, David A., "Strabismus", American Journal of Nursing, 74:1082-1089.

Greenslade, T. B. and Green, M. W., "Experiments with Stereoscopic Images, Phys. Teach. 11, 215 (1973).

Gregory, R. L. The Intelligent Eye (McGraw-Hill, N. Y. 1970).

Cravat, H. R. and Glaser, R., Color Aerial Stereograms of Selected Coastal Areas of the United States, U.S. Gov. Printing Office (1971).

The Pinhole Camera

PURPOSE:

To construct and use a pinhole camera for photography.

INTRODUCTION:

This laboratory experiment is designed to give you a cheap thrill! All you need is a discarded, inoperative polaroid camera (or an inexpensive polaroid camera), a roll of heavy-duty aluminum foil, sharp needles of various sizes, a comparator, and polaroid film. Assembled as a pinhole camera you have at your disposal a most humble imaging device which can do wonders as it creates unusual and even dramatic effects with today's film emulsions.

Find a beautiful model (yea! yea!) or a lonesome cow in the undulating fescue meadow (for you farmers!) or even perhaps the sun setting in the solitary oak tree on boot hill (for you melancholy poetry lovers), and shoot-away as they say in the "westerns".

CONSTRUCTING THE PINHOLE CAMERA:

You can make a pinhole camera with a light-tight box and film sheet; however, it is easier and the photographs are more reporducible if you use a polaroid colorpack camera with the lens and shutter removed. With lens and shutter removed, the square opening can be covered with heavy gauge aluminum foil and taped in place with black pastic tape. Be sure to check closely for light leaks by holding up the camera to the light and looking inside. After the foil is taped in a place, a pinhole is made in its center with a careful prick from the point of a needle. It is a good practice to place a piece of black felt over the entire foil and tape it along the top edge so that it can be used as a shutter. The black felt may be held over the pinhole with the hand or you may use a small piece of tape at the bottom of the felt to hold it down between exposures. Remember your pinhole camera must be very still while you are taking a picture. Either use a rigid tripod or hold steady or perhaps tape your pinhole camera to a table, windowsill, chair, rock, or some other rigid surface.

EXPERIMENTS:

1. Optimum Exposure Time

Of course the exposure time for the film is a function of the light present. For example, film that would take three seconds exposure time in bright sunlight might need seven seconds exposure time in cloudy, bright light for optimum exposure. Again, as with optimum pinhole size, you will attack the problem with the trial-and-error method.

Using a pinhole size of 0.25 to 0.35 mm, try exposure times of 5 seconds, 10 seconds, and 15 seconds using 3000 speed film. Which is the optimum exposure time?

2. Optimum Pinhole Size

If the aperture, i.e., the pinhole, is too large the image on the film will be blurred because the light from the subject is projected along several pathways causing a series of slightly displaced multiple images.

If the pinhole is too small, diffraction patterns will occur at the edges causing a significant degradation of the image resolution, i.e., sharpness.

To find optimum pinhole size it is recommended that you make a linear row of multiple pinholes of different sizes in the same foil and photograph the sun. Use approximately 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, and 0.7 mm size pinholes. You will find that sewing needles with gauge sizes no. 10 (measures approximately 0.45 mm at the center of shaft), no. 9 (approximately 0.56 mm), no. 8 (approximately 0.63 mm), no. 7 (approximately 0.69 mm) or beading needles of approximately 0.30 and 0.35 mm will provide the proper pinhole sizes. It is further recommended that you actually measure the pinhole size with a comparator because the hole size will be a function of how far you push the needle point into the foil. Note that since the light passed by cam be used without the customary focus correction required for cameras using lenses. As a result there is a total lack of chromatic aberration.

The pinhole camera has no focal plane. This is not true for a camera with a lens. With a pinhole there is a continuous image plane which allows the film to be exposed any distance from the pinhole. This will allow you to enlarge any portion of the image by simply moving the pinhole further away from the film. The image size will increase with an increase in the pinhole-to-film distance.

EQUIPMENT NEEDED:

1. Polaroid camera (approximately $30.00) colorpack model.
2. Black and White polaroid film (approximately $3.00)
3. Sewing needles (sizes no. 7, 8, 9, 10 and beading assortments)
4. Heavy duty aluminum foil
5. Black plastic tape
6. Black felt cloth
7. Comparator (approximately $24.00)

REFERENCES:

Kodak Customer Service. How To Make and Use a Pinhole Camera, Publication no. AA-5, 1971.

Mims, Forrest M., "The Pinhole: A Lens That Just Won't Quit," Popular Photography, April, 1974, p. 101, 137-138.

Tolansky, S., Curiosities of Light Rays and Light Waves, American Publishing Company, Inc., 1965.

Young, M. "Pinhole Imagery," American Journal of Phsyics, Vol. 40, No. 5, p. 715-720 (May, 1972).

Young, M., "Pinhole Optics," Applied Optics, Vol. 10, No. 12, p. 2763-2767 (Dec. 1971).

Technical Notes

The lens and shutter can be completely removed from the Colorpack camera by removing three screws. The camera back is then available for holding the polaroid film pack. The camera front comes off as one piece and can easily be reassembled on the back if desired.

With a pinhole diameter of 0.25 mm the exposure time with 3000 speed film is approximately ten seconds. Exposure time will vary reciprocally with the square of the pinhole diameter.

Remember that as soon as one film sheet is removed from the camera, the next is ready for exposure, so the pinhole must remain covered until you are ready.

TELEVISION OPTICS

Objective

The objective of this experiment is to demonstrate some of the capabilities and limitations of television systems in the formation of images and reproduction of colors.

Equipment Required

A monochrome (black and white) television camera with zoom lens, a color television camera with zoom lens, color television monitor, slide projector, diffraction grating for projector, printed spectrum chart, flood lights.

Procedure

Imaging:

The television camera can be likened to an ordinary camera in which the film has been replaced by the television photosensitive tube. The image is seen immediately on the screen of the monitor (or receiver) instead of on a photographic transparency or paper. For our purposes here, the immediacy of television is a great advantage. It allows us to see what image is being formed by the system without waiting to process the film.

The Zoom lens:

The zoom lens is a combination of lens elements which can be moved relative to each other to allow a continuous change of the effective focal length of the lens. It allows one lens combination to perform the function of several different lenses without the necessity of changing lens.

Lenses are characterized by their focal length. A small focal length lens gives a wide field of view and is called a wide angle lens. A large focal length lens gives a smaller field of view and is known as a telephoto lens.

Set the camera lens at the position of greatest focal length. Aim the camera at an object across the room and focus the image. Do not alter the zoom (or focal length) while adjusting for sharp focus. Without changing the camera position or focus rotate the zoom control slowly to gradually change the effective focal length to its smallest value. Then aim the camera at something much closer to you. Center the subject in the picture and slowly return the zoom to the telephoto position. Notice what is happening. When you have reached the maximum, refocus to again get a sharp image.

The depth of field is the range of distances from the camera that an object is in focus for a particular lens and focus adjustment. Measure the depth of field of the zoom lens at maximum, minimum and midway between settings of the effective focal length. Do this for best focus distances of 2 meters and 4 meters.

Aperture and Lighting:

If you can do so, vary the light level from just enough for the camera to operate to very bright. Watch what happens to contrast, sharpness, and depth of field. These effects are due to the decreasing size of the aperture with increasing light. A smaller aperture reduces the area of the lens that is being used and minimizes the abberations which are inherent in images formed with lenses.

Color:

Make a slide which is opaque except for a central slit 5 to 10 mm wide. With a slide projector image the slit on the screen. Place a good diffraction grating right in front of the projector lens and adjust it to give a good spectrum on the screen. You will need a very bright projector and a very efficient grating in order to have enough light for this experiment.

Look at the spectrum with the color television system. Compare what your eye sees with what the camera sees and reproduces.

Next, look at the printed spectrum with the color camera. Compare how it looks to your eye with the television picture. Vary the intensity of the illuminating light to see what effects that has on the television picture.

Repeat the observations of the real and printed spectra with the monochrome camera. If possible use the monochrome camera to examine the monitor image generated by the color camera.

Questions

1) When photographing or televising a scene in which the focal length of the lens will be changed, why is it good practice to zoom in close (telephoto setting), focus, and then zoom back to wide angle before starting? What happens if the focus is set from the wide angle position?

2) How does restricting the aperture affect the depth of field?

3) What colors of light does the color television camera respond to? The monochrome camera?

4) How does the color television system reproduce colors in the light reflected from objects, especially colors that are not well reproduced from the spectrum?

Technical Notes

The spectra required in this experiment need to be very bright in order to be seen by the television cameras. We have been successful by using a lantern slide projector with a 1200 watt lamp as the source. The slit width of 10 mm is adequate. The grating was from the Education Absorption Spectra Kit Manufactured by Bausch & Lomb, cat. no. 33-80-10 (approximately $50.00). This grating provides about 200 times the intensity of the cheap (25¢) gratings.

A satisfactory printed reproduction of the spectrum is available from Kodak for $1.00. Ask for spectrum reprint H7-42.

If the television equipment is unavailable, or as a further inquiry, this experiment can be carried out with a super 8 movie camera and projector. If this is done it is instructive and interesting to compare different films. The spectral responses of Kodachrome II (KA 464) and high speed Ektachrome 160 (ELA 464) are quite different and are interesting to compare.

Light Sources and Color

Purpose: To observe changes in the colors of objects when they are illuminated by a variety of light sources

Introduction: Color pigments reflect some wavelengths (colors) of light and absorb others. The reflected wavelengths all combine to yield the observed color of that object. A white light source emits all the colors of the spectrum. Sunlight and most artificial light sources give a full spectrum of colors. The relative intensity (brightness) of each color depends upon the kind of light source. Some other types of sources do not emit a full spectrum. Using a grating spectroscope - which is an instrument that diffracts each color of light through a different angle- one can see the color spectrum emitted by the source being observed.

The relative intensity of each wavelength of light reflected by an object and, therefore, its color depends upon the character of the spectrum of the light illuminating it.

Candle light is deficient in blue and violet light; so an object illuminated by a candle will reflect relatively less blue and violet than red and yellow; and the object will have a "warmer" color.

Neon gas lights, sodium vapor lamps, and mercury vapor lamps do not have a continuous spectrum of all the colors. They have only a few bands of color in their emission spectra.

Fluorescent lights have a bright band spectrum combined with a continuous spectrum from the phosphor powder inside the tube.

The human eye adjusts to these color shifts to some extent, and they are not always noticeable. Color film is more sensitive to these color shifts and allows one to compare directly the color of an object photographed under different light sources. The simplest way of indicating these color shifts is the concept of color temperature.

Apparatus: Polaroid camera (Colorpack II, approx. $25), Polaroid color film (approx. $4), meter stick, simple grating spectroscope, several light sources (sunlight, fluorscent light, incandescent bulb, photoflood lamp, candle, mercury or sodium vapor lamp, etc.), scissors, large painting or print with a full range of colors(Polaroid camera box has a nice spectrum chart).

Procedure: Using the Polaroid camera and color film, photograph a large painting or an arrangement of several small paintings that have a full range of colors. Make these photographs with different sources of illumination. Be very careful to eliminate light source glare as seen by the camera. Use the grating spectroscope to observe the color content of each light source. Trim the white border from the edge of the prints to eliminate a white reference source. Label each photograph and compare the effect for each light source.

Questions: How does the color content of a light source change the colors one observes? What colors are lost and what colors are enhanced by each source? What light sources are best for an art gallery? Can one determine (predict) how colors will appear when illuminated by a particular light source by studying the spectrum of that source? Is there a correlation between the "temperature" of the light source and the overall color shift of the photographs?

Notes to the Instructor: Polaroid Colorpack II camera can be used in this experiment and also in the pinhole camera experiment. It involves the removal of only three screws and is easily put back together. Discount stores usually have these cameras for about $10 less than camera stores.

The fluorescent lights already in the lab are the best source of fluorescent light. A slide projector, without a slide of course, is also a good source. If necessary, you can wait until dark and use the mercury vapor lamps outside. The exposure for dim light sources will be long in time so a rigid support is needed for the camera during the exposure.

Hints for Obtaining Good Photographs: Set correct lens distance for proper focus. Set film speed on camera to 75. For low level illimination, use a rigid support for the camera to avoid blurred photographs. Adjust the light sensor on the front of the camera to lighten or darken the photograph for correct exposure. Do not cover the light sensor or allow the light source to shine directly into the sensor.

References: "Colour, Its Principles and Applications" Frederick W. Clulow, Fountain Press 1972.

"Colour Science" W.S. Stiles and G. Wyszecki, Wiley 1967.

"Color as Seen and Photographed" Kodak pamphlet #E-74 $1.

Kodak pamphlet #H742 $1.

Extra Project

A blazed grating (Bausch and Lomb Cat. No. 33-80-10, $47) and a projector can be used to produce a brilliant spectrum.

I. Make a slide that has a single slit approximately 5 mm. wide and insert it into the projector. Place the grating in front of the lens and project the spectrum onto a screen or large white surface. Use the Polaroid camera to photograph this spectrum.

Questions: Why are only three colors recorded by the photograph? How is it possible for the film to record a yellow object as yellow even though the film does not detect yellow wavelengths of light?

II. Take three small front surface mirrors and place them in the spectrum. Use them to reflect a particular color of the spectrum onto a screen or white sheet of paper.

Use two or three mirrors at the same time and overlap three reflections to generate other colors.

Questions: Can you get all the secondary colors? Can you get white? How many colors must be added to get white?

AUDIOMETRIC MEASUREMENTS

EQUAL LOUDNESS CONTOURS

The purpose of this experiment is to plot equal loudness (Fletcher-Munson) contours for your ears. Note that the curves presented in textbooks are "typical" curves, but your own psychophysical response may be different.

Procedure:

1. Seat yourself about 5 or 6 feet from a loudspeaker with your back to the speaker. Place a sound level meter directly behind your head so that it can be read by your partner. Have him provide you with a 1000 Hz tone of 50 db (on the C scale).

2. Have him alternately switch from the 1000 Hz tone to a 500 Hz and adjust the latter tone louder or softer until it appears to have the same loudness as the 1000 Hz tone. Measure the Sound Pressure Level and record on a graph. Repeat at frequencies of 60, 100, 200, 2000, 5000, 10,000, and 15,000 Hz. Draw a smooth curve representing the 50 db loudness contour (points on this curve are said to have a loudness of 50 phons).

3. Repeat for SPL's of 30, 70, and 90 db.

4. Change places with your partner and help him make a similar graph for his hearing. Compare the two graphs.

PURE-TONE HEARING THRESHOLD

A clinical audiometer, such as the Beltone audiometer, is most suitable for this part of the experiment, but in the absence of this, a pair of head phones connected to an oscillator via a switch can be used. A V U Level meter is also required. Most tape recorder amplifiers have such meters, which can be used in series with the oscillator. The headphones should be the best acoustic type, which completely enclose the ears, and reduce ambient noise, and the subject must be placed in a low-noise environment. Furthermore, it should be noted that the headphone - head enclosure is resonant, as can easily be seen by varying the oscillator frequency at constant amplitude, when the subject will hear a series of loud-soft tones. This may be avoided by using loudspeakers in a small anechoic chamber, but otherwise, try and avoid resonant frequencies. The switch should be quiet, such as a mercury switch, and a large resistance may be necessary in series with the headphones to reduce the intensity to a low enough value without pickup or hum being audible.

Procedure:

There are various psychophysical techniques that might be used for determining thresholds with a manual audiometer, but the technique recommended for these measurements is described below:

1. Give instructions to the subject regarding the test procedure so that he knows what to expect and how to respond.

2. Set the test tone selector at 1000 Hz and the hearing level control to its lowest setting and the tone switch to the normally off position.

3. Place the earphones carefully over the subject's ears.

4. Slowly increase the hearing level control while holding the tone switch down (tone on) until the subject responds by raising his finger.

5. Turn the tone off and on again to be sure the subject is responding to the test tone.

6. After this level is established, lower the hearing level control 10 dB with the tone off and, at this point, use the tone switch to present a 2-second tone to the subject. If the subject does not respond (a "nay" response), increase the hearing level 5 dB and present the 2-second tone

again. The subject may respond at this point (an "aye" response). Decrease the level 5 dB, where the subject should repeat a "nay" response. Once more increase the level 5 dB, where an "aye" response would complete a series of 2 "aye" and 2 "nay" responses. The pure-tone threshold is taken as the hearing level control setting for the "aye" response. If the subject does respond at the beginning of this step when the hearing level control is lowered 10 dB, the level should be reduced in 5 dB steps until there is no response, and then repeat the threshold crossing procedure to establish the "aye" response threshold.

7. Repeat for test tones at 125, 250, 500, 2000, 4000, and 8000 Hz.

8. Make a graph of threshold as a function of frequency. Is there evidence for significant hearing loss?

APPENDIX

RELEVANT REFERENCES FROM THE AMERICAN JOURNAL OF PHYSICS

1. Musical Combination Tones and Oscillations of the Ear Mechanism - Olson D 37/7/730 (69).

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23. Surface Vibrations of Liquids - Ramanadham M 34/6/538 (66)N.

24. Demonstrations of Waves on an Air Track - Howland L P 33/4/269 (65).

25. A Sound Pipe - Daw H A 36/11/1022 (68)N.

26. Acoustical Interferometer for 'Open-Ended' Laboratory Experiments - Boyer R A 34/10/946 (66).

27. Experiment for the Study of Acoustic Wave Phenomena - Wright W M 38/1/110 (70)N.

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36. Beam-Scanner for Experiments in Optics, Acoustics, and Microwaves - Syed B A 35/6/538 (67)N.

37. Simple String Driver - Nahshol D 33/10/856 (65)L.

38. A Determination of the Speed of Light by the Phase-Shift Method - Rogers J, McMillan R, Pickett R, Anderson R 37/8/816 (69).

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Appendix 2

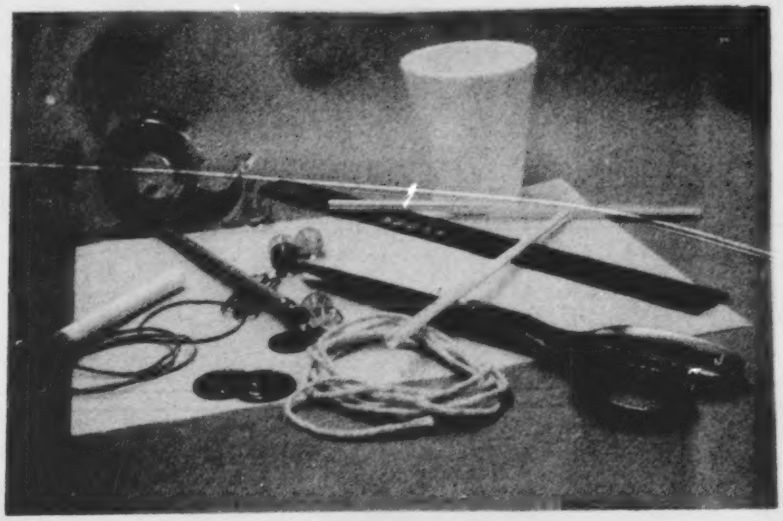
Simple Experiments

Introduction

Even in this age of atom bombs and rockets, schools still have difficulty finding experimental equipment for their students. The following experiments were put together to see what could be done with the simplest, least expensive materials. The equipment can all be purchased at the nearest dime store - not even a stop watch is required, nor are you exhorted to "go down to the junk yard and pick up a 2000 volt transformer" as is done in some books requiring economy. In spite of their simple nature, the experiments are quite meaningful, and demonstrate fundamental physical laws in a practical way.

Equipment

The experiments involve only common rubber bands (about 3" x 1/16", if available), cellulose tape (the cheapest, clear half-inch kind), regular paper clips (#1 are best), styrofoam or Dixie cups, and drinking straws (plastic, and preferably translucent), glass marbles, paper, blackboard chalk, a foot ruler with a groove down the center, provided to prevent the pencil rolling off the desk, coins, a pencil and scissors. It is a good idea to collect these simple items and keep them in a box, so that they are easily available.

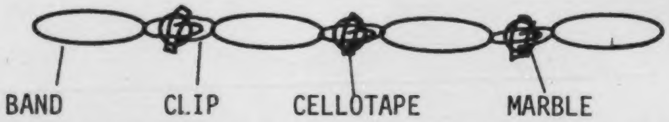


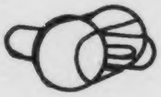
I am indebted to Ronnie Edmonds for the diagrams

Experiment A50: Longitudinal Waves

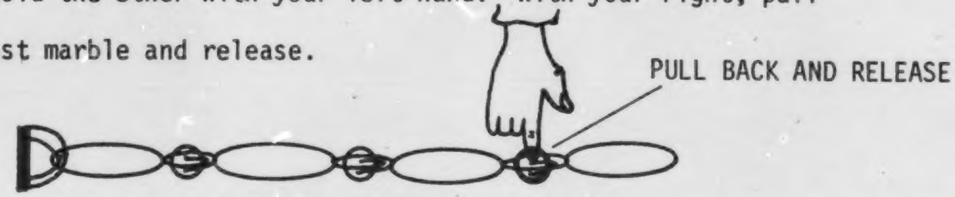
Materials: - rubber bands, marbles, paper clips

Instructions: - The aim of the experiment is to construct a device along which longitudinal waves travel slowly, so that the motion may be followed in detail. Connect sixteen paper clips in a string using sixteen rubber bands, to provide a weak restoring force. To slow the longitudinal wave, attach two marbles to each paper clip with cellotape.





Now, attach both ends to a firm anchor - you can fasten one end to your desk, and hold the other with your left hand. With your right, pull back the last marble and release.



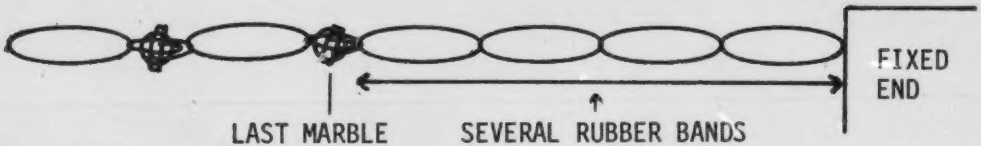
Watch the compressive pulse travel along and be reflected. We call it compressive because each marble moves in the direction the wave travels, pushing the one ahead. After reflection; is the pulse compressive? It is often difficult to follow the pulse down the string, but if you watch the end marble closely, you will see it jerk backwards and forwards each time the pulse passes.

A rarefaction occurs where the marbles move in a direction opposite to that in which the pulse travels - so if you move the marble away from your left hand before releasing, you get a rarefaction traveling down the system. Sound waves in air are composed of successive compressions and rarefactions. You have looked at reflection from a fixed end. Such reflections occur with sound waves in organ pipes closed at one end. Standing waves are built up in such pipes. You can simulate such a standing wave by moving the hand holding the rubber band backwards and forwards until you hit a resonance. The marble

L 2

near the far end must be stationary. This is called a node. The marble in the middle moves rapidly, which is called an antinode.

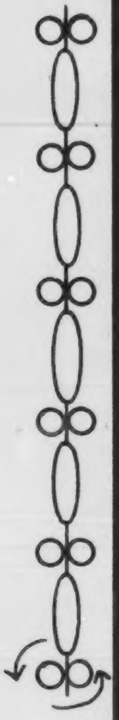
To examine what happens if we have an open organ pipe, attach three or four rubber bands without marbles or paper clips between the far end of the string and the table.



Now, if you feed in a compressive pulse, is it reflected as a compression or a rarefaction? Try producing standing waves You will find the end marble, which was stationary, now moves more than all the rest - so what was a node for a closed pipe, is an antinode for an open one. This arrangement can also be used for transverse waves, but the soda straw device, mentioned elsewhere, works better.

Torsional Waves

If the device is hung vertically from a support, torsional waves may be generated by rapidly twisting the lowest rubber band by rubbing it between the thumb and forefinger. The bottom clip and marbles spin rapidly, and pass this motion very slowly to the top of the chain. Here the pulse reverses and the marbles spin in the opposite sense. Reaching the bottom, which reflects like an open ended pipe, the marbles continue to spin, and wind up in the same sense, the pulse again traveling up the chain and reversing at the top. This proves a dramatic demonstration of the difference between reflection at an open and closed end.

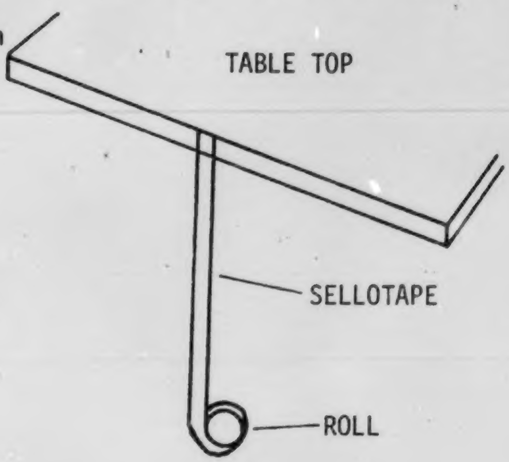


T 1

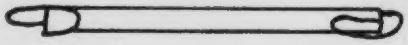
Experiment 45 Transverse waves

Materials - Cellotape, about two dozen drinking straws, paper clips.

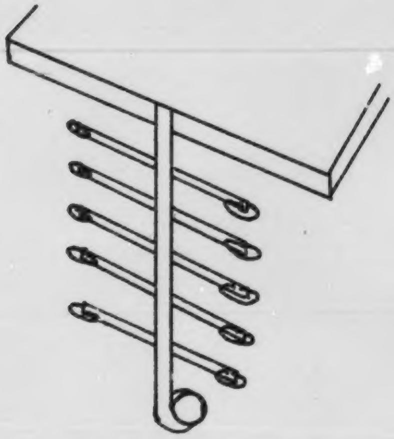
Procedure - Attach one end of the cellotape to the table top, pull about two feet off and let it hand down



Place one paper clip in each end of each drinking straw



Stick the center of the straw at one inch intervals along the cellotape, until you



have about 24 of them attached. Now, looking end on at the straws, pull the cellotape reel, to make the striptant, and give the bottom straw a tap. You will see a transverse wave.

T.2
5/

pulse travel up the strip, and be reflected at the top

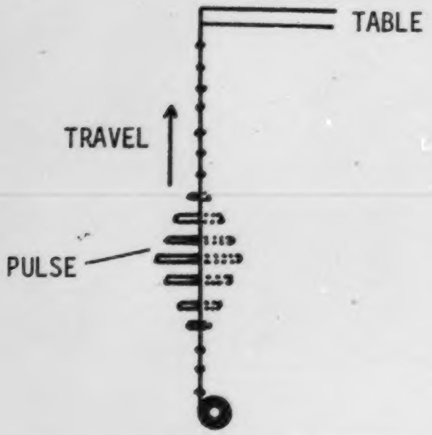
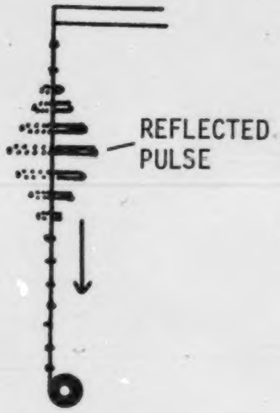
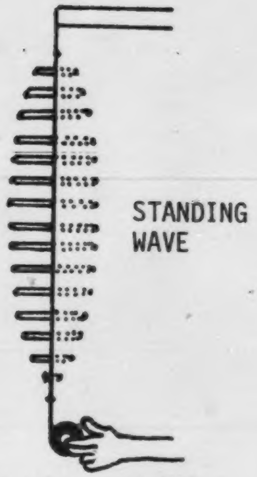


TABLE
TRAVEL
PULSE


REFLECTED
PULSE


STANDING
WAVE

You may induce standing waves by rotating the bottom straw too and fro with the right period. If you unreel a length of tape, you may study reflection from a free end, just as you did reflection from a fixed end

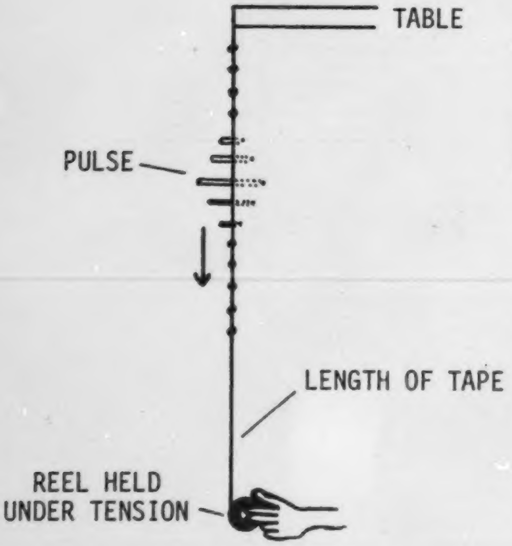
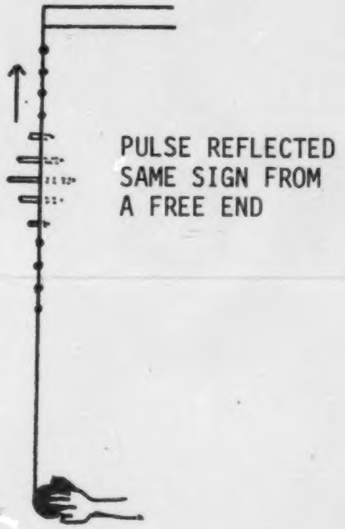


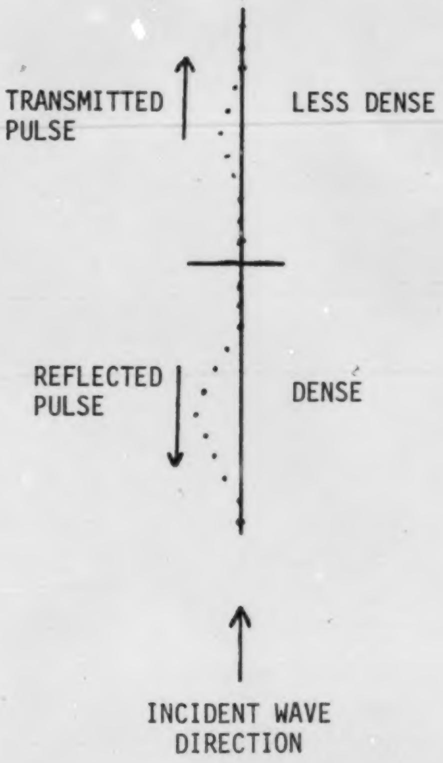
TABLE
PULSE
LENGTH OF TAPE
REEL HELD
UNDER TENSION


PULSE REFLECTED
SAME SIGN FROM
A FREE END

For the last foot or so of the tape, put two paper clips at each end. Now you can study the reflection of a wave traveling from a less dense to a denser medium (top to bottom) or vice versa (bottom to top). Note how, in each case, part of the wave is reflected at the intersection; but in one case it changes sign (phase)

T 3
5/

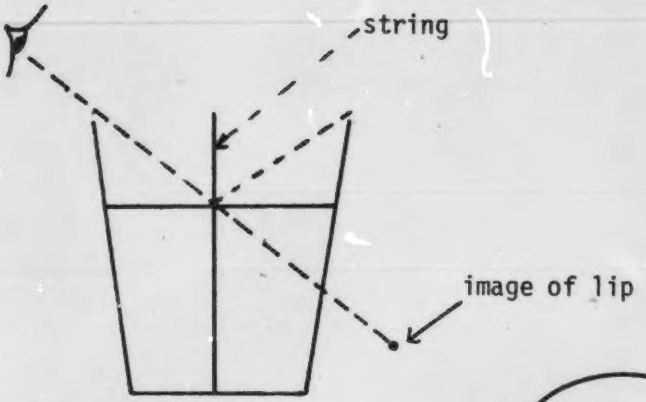
and in the other case it does not.

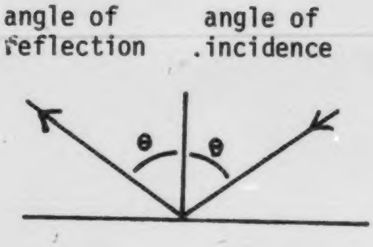

TRANSMITTED PULSE
LESS DENSE
REFLECTED PULSE
DENSE
INCIDENT WAVE DIRECTION

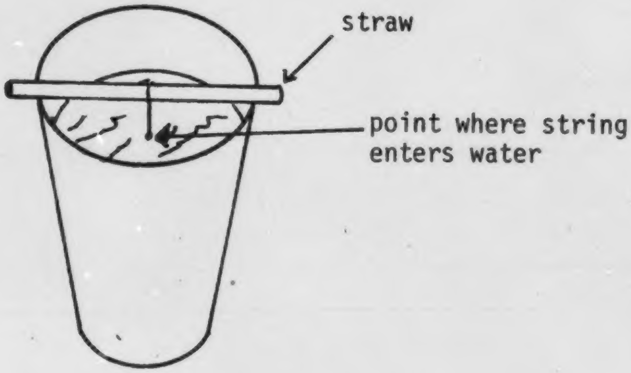
Experiment A5: The Law of reflection of light.

Materials: - Two dixie cups, soda straws, string, cellotape.

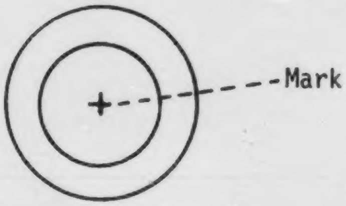
Instructions: -


string
image of lip


angle of reflection
angle of incidence
θ
θ


straw
point where string enters water

Make a mark, as close to the center of the Dixie Cup at the inside of the bottom as possible.



Make a small hole, tie the string around the center of the straw and pass the string through the hole, pulling and taping over the bottom, so water will not leak out. Adjust the straw so that the string is exactly vertical. Now pour a little water in the cup, and look at the reflection of the far lip. . Lower your eye until the

L2

reflection of the far lip is exactly on the rear lip. See if the string enters the water at this point. Examining the figure, you can see that if this is so, by symmetry, the angle of incidence on the water is equal to the angle of reflection. You can tilt the cup a little to adjust until where the string enters the water does lie exactly on the near lip, and the reflection the far lip. Now, pour in some water, and repeat the experiment. This increases the angle of incidence. Repeat for several angles of incidence.

Qualitative Questions: - What does this show about the reflection of light?

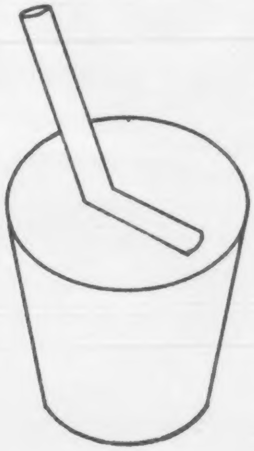
Quantitative Questions: - With what accuracy have you shown the angle of incidence is equal to the angle of reflection? (estimate this from the tilt you gave the cup) 10%? 1%? or 1/10%?

Experiment 56 The refractive index of water

Materials - Dixie cup (as deep as possible), pencil, ruler

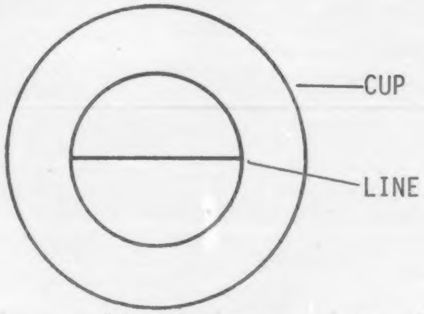
Procedure -

Qualitative - Place a pencil in water in the cup. Notice it appears bent as it enters the water. This is because the light travels more slowly in the water, and rays of light are bent as they leave the water surface.



Quantitative : Draw a line straight across the middle of the dixie cup inside. Fill the cup to the brim with water.

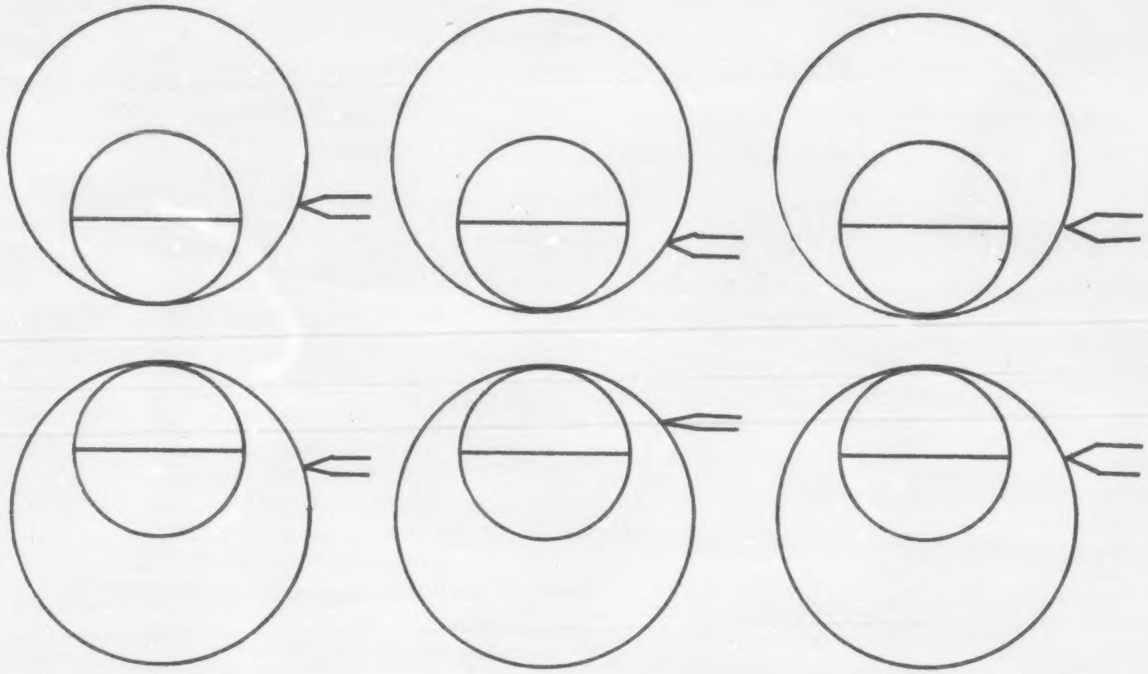
SEEN FROM ABOVE



Place a pencil point against the outside of the cup where the line appears to be and move the head up and down. Adjust the pencil until it is at the apparent depth of the line.

Make a mark on the side of the cup. Measure the distance from the lip to the mark, and to the bottom where the line was drawn.

SEEN FROM ABOVE



PENCIL TOO HIGH

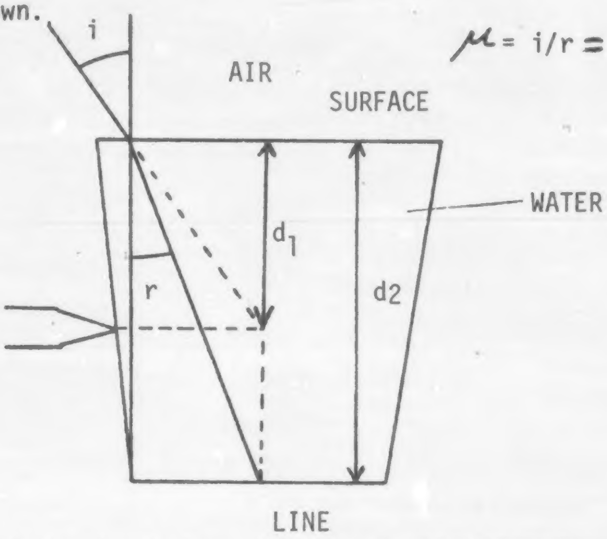
PENCIL TOO LOW

PENCIL JUST RIGHT
(NO PARALLAX)

What we learn

The refractive index is the ratio of the sine of the angle of incidence to the angle of refraction. For small angles, this becomes the ratio of the angles, as shown.

$$\mu = i/r = d_2/d_1$$



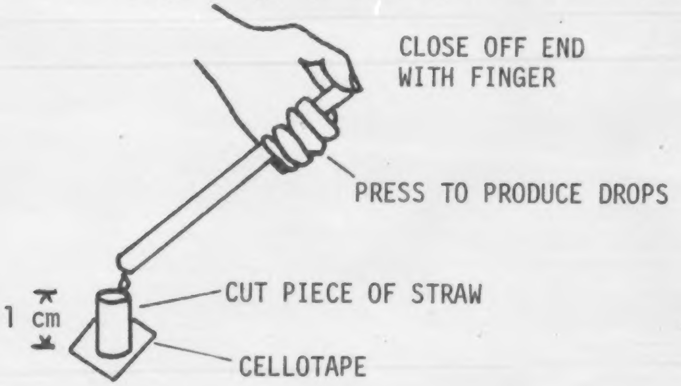
Hence, the ratio of the depth from the brim to the line drawn in the cup, to the depth of the pencil, is equal to the refractive index, which is 4/3 for water.

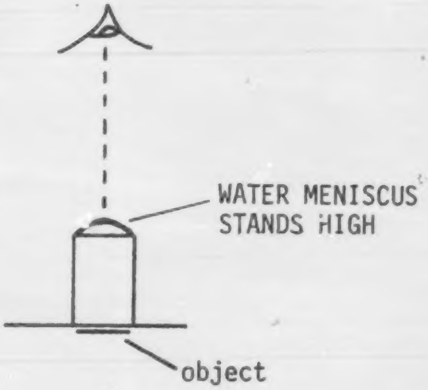
57.

Experiment 57 - Optics - Positive and Negative Lenses

Materials - Cellotape (clear), soda straw, water

Procedure - Stick a piece of cellotape flat over the end of a soda straw and cut a piece 1 cm (about 1/2 inch) long from that end. Place the cellotape over some object - a fly, or a printed letter - and fill the piece of soda straw, using the longer piece sucked full of water.



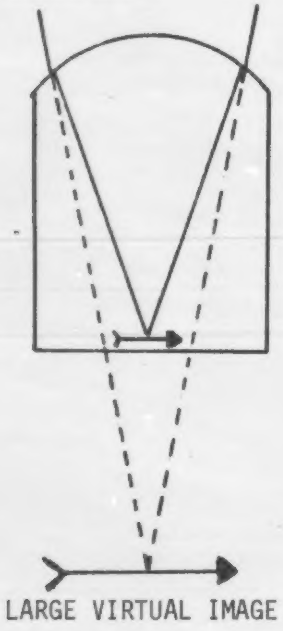


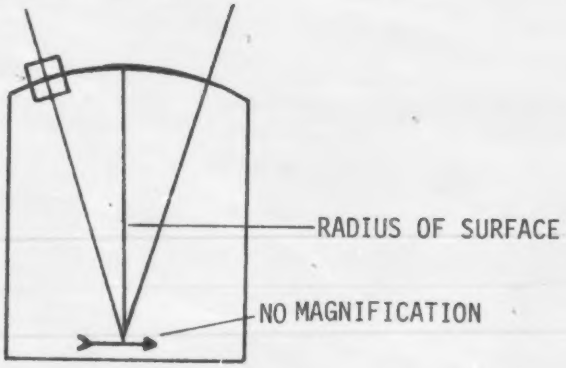
Make sure the meniscus stands high on the straw. Now, look down through the straw. You will see a magnified image of the object underneath. As the water leaks out of the bottom (or, you can soak up a little with toilet paper) the meniscus changes shape. When it stands high, we say it is convex. Hence, a convex lens can magnify in the same way as a magnifying glass, producing an image larger than the object. As the water leaks out, the surface caves in, and we say it is concave. Look at the object through the water now, and you see it appears much reduced in size like looking through the wrong end of a pair of binoculars. Concave lenses therefore give an image reduced in size. What do we learn?

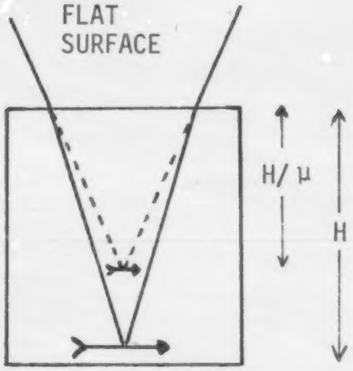
Qualitative - Convex lenses (bowed out) magnify

Concave lenses (bowed in) give images reduced in size

Quantitative - The ray trace of the system is shown below


LARGE VIRTUAL IMAGE


RADIUS OF SURFACE
NO MAGNIFICATION


FLAT SURFACE
H/ μ
H

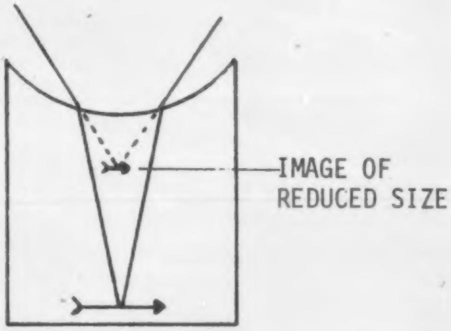
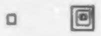


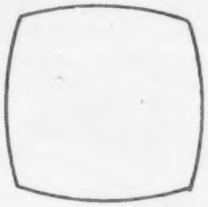
IMAGE OF REDUCED SIZE

μ - the refractive index. the ratio of the speed of light in air to that in the medium.

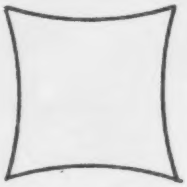
Look at the little square below through the water lens. When the meniscus is convex



it looks like



This is known as barrel distortion, because the square image is distorted to look like a barrel. Distortion of this kind occurs with all lenses having spherical surfaces, such as this. When the lens becomes concave, the distortion changes and becomes pincushion distortion, so



Experiment - Real Images

Materials - As for previous experiment, straw, water

Procedure - Take the water lens from the previous experiment, and fill it until the surface is convex (bulges out). Now, hold the lens vertically under the room light, a few inches above a sheet of paper. Move the lens up and down until you get an image or picture of the light on the sheet of paper. Because it actually lies on the paper, this is called a real image. The image is distinct, but not very clear. Notice as water leaks out, (or if you soak up a little on toilet paper) the image gets farther away from the lens. We say the focal length (the distance from the lens where a point very far away focuses) is increasing. Notice also that, as this occurs, the image gets bigger.

What we learn

A convex lens can form a real image, as shown — In the case of our water lens, where the bottom surface is flat, we can calculate the radius of the top surface from the formula for a thin lens-

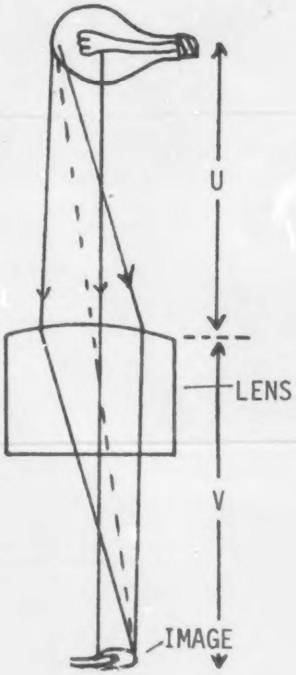
$$\frac{1}{u} + \frac{1}{v} = (\mu - 1)\frac{1}{r}$$

where μ = refractive index of water = 1.33

u = distance of object from lens

v = distance of image from lens

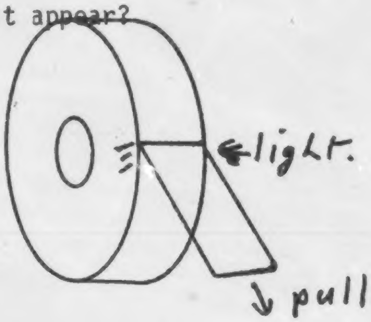
r = radius of curvature of the water surface



Experiment 158: - Energy Conversion Triboluminescence

Materials: - Cellotape

Take a roll of cellotape into a very dark room. Rapidly pull a little off the roll. Where the tape pulls off the reel, light is given out. However, it is a weak source, and for such sources, the outer regions of the eye are more sensitive, so do not look directly at the roll, but about a foot or two away from it. What color does it appear?



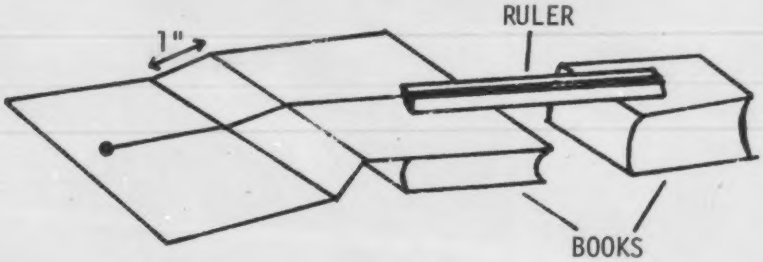
Qualitative: This effect is known as "triboluminescence" see "The Flying Circus of Physics" 6.11.

The bluish color arises because the rods of the eye are more sensitive to blue light, so all dim illumination appears bluish, since the rods are very sensitive to weak light.

EXPERIMENT A59 Refraction of particles

Materials: Chalk dust, cardboard, marble, ruler, protractor (from expt. 2)

Method: Fold about 1" down the middle of the sheet of cardboard, as shown, and place the top part on a book. Dust chalk over the surface.



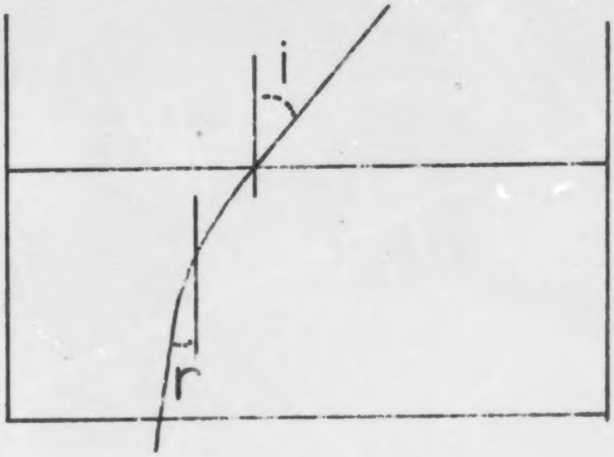
Now roll the marble over the dust, down the ruler, from the same height, but in different directions.

Qualitative: Does the particle bend in the same way light would in going from air to water? Do you think light could be a particle, like the marble?

Quantitative: Snell's law states

$$\frac{\sin i}{\sin r} = \text{const.}$$

Is this true for the trajectory of the marble?

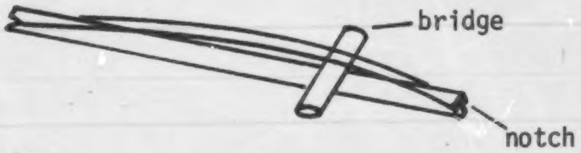


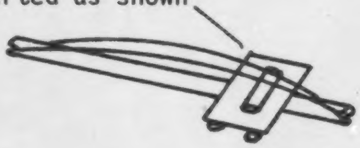
6

Experiment A 60 the rubber band guitar

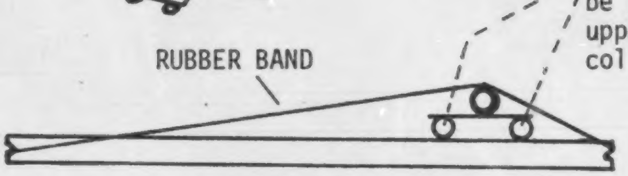
Materials: - soda straw, rubber band, cardboard

Instructions: - stretch the rubber band over the straw as shown (the ends of the straw may be notched). A small piece of straw acts as a bridge. A piece of cardboard may be inserted as shown


bridge
notch

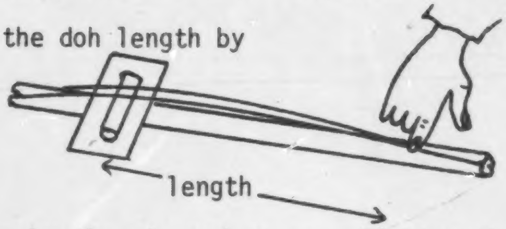


side view


RUBBER BAND
These supports should be pushed in until the upper support does not collapse the card

Does the guitar sound louder with or without the card in place?

Starting with the open band as "doh", place your finger on the band to shorten it, and mark on the straw the position of the notes of the octave. Are they equally spaced? Measure the distance of the marks from the bridge. Now divide the doh length by the ra length, the ra by the mi, and so on. Tighten the rubber band a little. Does the pitch go up or down?



Note: If you stretch a rubber band between your fingers and pluck it, it may go up or down in frequency, or even stay the same, as it is stretched. Think how the frequency depends on length, tension, and mass per unit length of the string to explain why this is.

Qualitative: - You have seen how the pitch of a plucked string depends on its length. How much shorter must a string be to give the octave? The pitch also depends on the tension on the string.

Quantitative: - The ratio of lengths, for successive notes, starting with do, should be ·89, ·89, ·95, ·89, ·89, ·95.

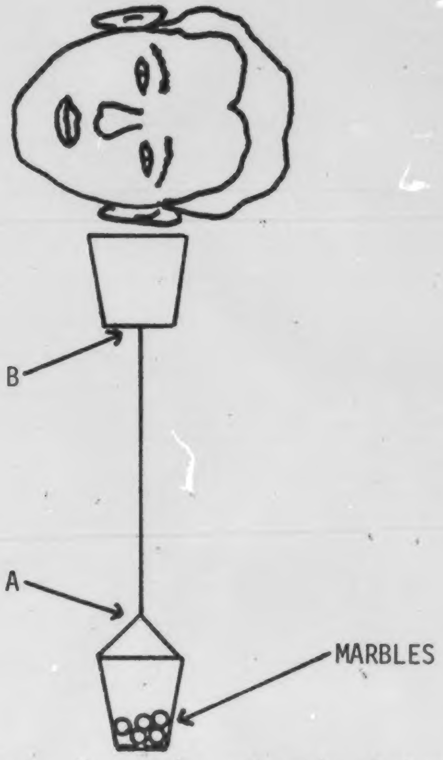
You will find this also on the guitar, notice that the big gaps are where the semitones (black keys on a piano) fit in. If you put in the semitone, the ratio is always ·95 (17/18) between successive notes, leading to a ratio of 2 for the octave.

Arrange the bridge half way along the string. The two halves give the same notes (unison). When the ratio is 2:1, the octave is heard 3:2, the fifth and 4:3 the fourth. Such ratios are pleasing to the ear, and called consonants. Pythagoreans thought these ratios had a mysterical significance.

Experiment N 1: The Way the Tension and Length of a Plucked String Affect its Pitch.

Materials: Two styrofoam (or Dixie) cups, string, marbles and a ruler.

Procedure: Hang one cup from the string, as shown, passing the string through holes poked in the top on opposite sides, and pass the other end of the string through a small hole poked in the bottom of the other cup. Tie small knots in the string.five inches, ten inches, and twenty inches from A. Put ten marbles in the cup and pluck the string. Hold your ear over the cup, and you will hear a clear tone, the string vibrating between A and B. Now, pull the string up until the 5 inch knot is at B, and pluck again, noting the pitch. Drop to the 10 inch knot and pluck again, and the 20 inch knot. Does the pitch drop an octave in each case? If it does, the frequency also drops by a factor of two each time, since it is known, and we must assume that a note an octave higher than a second note, has twice its frequency.

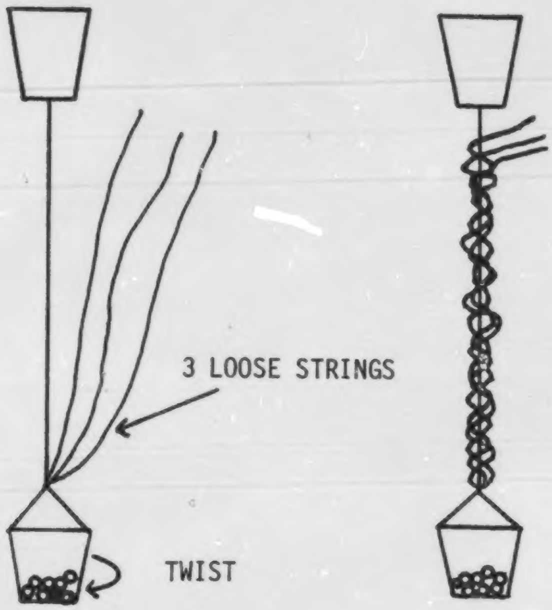


Now set the string at the 10 inch knot and pluck it. Add ten more marbles, and drop the string until it gives the same pitch as before. Measure the new length of string from A to B, using the ruler. You can go up and down, adding and taking away ten marbles until you get it right. What is the new length? Is it twice the old length? No, it is much less, showing that if you double the tension, the frequency goes up less than twice. In fact, the frequency should be proportional to square root of the tension. Now, we doubled the tension by putting in twice as many marbles, so the length of string should be $\sqrt{2}$ larger. $\sqrt{2}$ is 1·412, so the new length should be 14.12 inches.

Find the length having the same pitch for 10, 15, 20 and 25 marbles in the cup. Put the numbers in the table below. Do they agree with the calculated values? If so, you have shown that the square root of the tension is proportional to the length of string for the same pitch, and we deduce that, if we keep the string the same length, the square root of the tension is proportional to the pitch. So we need not twice, but four times the tension to make the pitch increase a factor of two, i.e. an octave. Try this out by putting as many marbles as possible in the cup, and then reducing to one quarter. Does the pitch drop an octave?

Number of marbles	$\sqrt{\text{number}}$	3.16 x $\sqrt{\text{number}}$	Experimental length, inches
10	3.1622	10	
15	3.8729	12.24	
20	4.472	14.142	
25	5	15.8	

What happens to the pitch with a thick string? Attach three strings to the top of the lower cup, with, say twenty marbles in. Pluck the string. Now, spin the lower cup. The three loose strings will wind around the one taut one, so



it is four times as heavy--the mass per unit length is four times as great. Again pluck the string. What happens to the pitch? Take out marbles until you have the same pitch as before.

Question: How many marbles must be removed? Is the pitch proportional to 1/mass /unit length or $\frac{1}{\sqrt{\text{mass/unit length}}}$

6–

Experiment A62. Waves in Pipes

Materials: Soda Straws

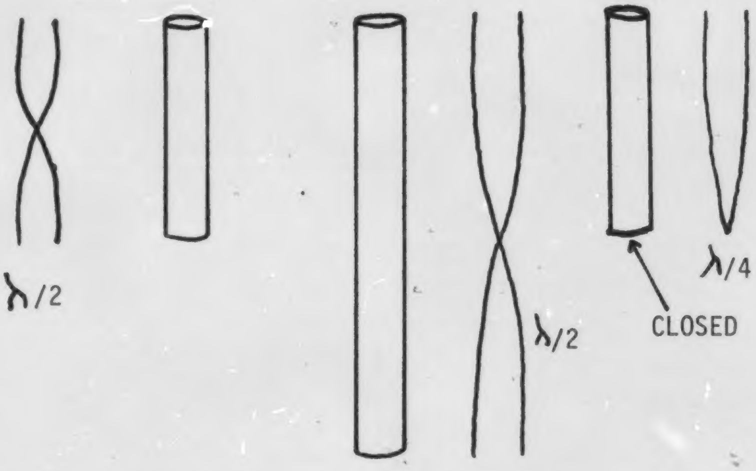
Procedure: Take two straws. Cut one in half. Blow across the end of each tube. How does the pitch of the longer tube compare with the shorter.

1. Does the pitch go up or down with the length?
2. Is a pitch change of an octave or a fifth given by a length of a factor of two?

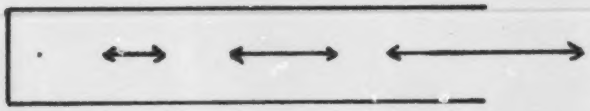
Close the bottom end of the smaller tube with one finger, and again blow across the top end.

3. Does closing a pipe raise or lower its pitch?
4. Does its pitch change by an octave? Is its pitch the same as an open pipe twice the length?

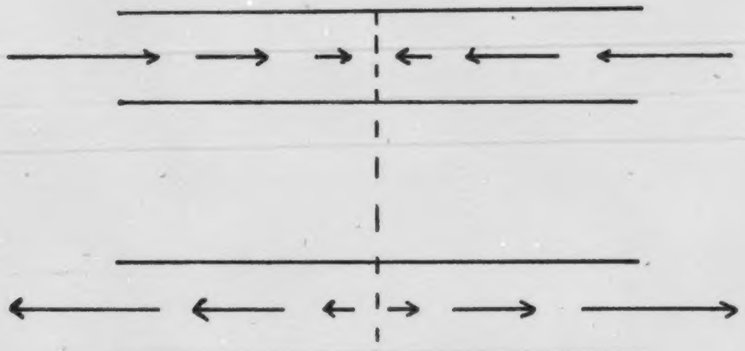
Why is this?



The closed tube has a node at one end, and its fundamental node is one quarter of a wavelength. In this node, air rushes in and out of the open end, the



motion diminishing until at the far end it is zero. The open tube has a node at the center, and is half a wavelength long.



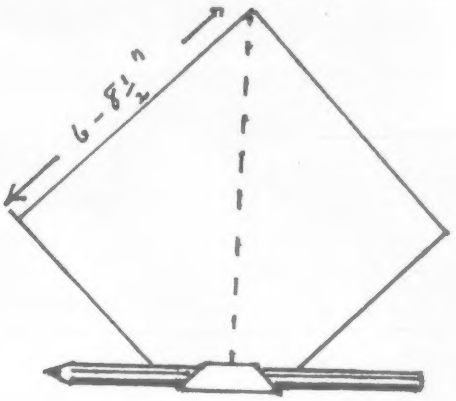
NODE

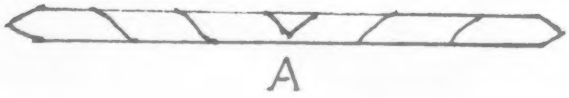
To determine frequency threshold of hearing, cut a straw into shorter and shorter lengths until the frequency can no longer be heard on blowing across the top. The upper frequency is about 16,000 Hz, which is produced by a straw .41 inches long, roughly half an inch.

Experiment A63: Musical Instruments using reeds.

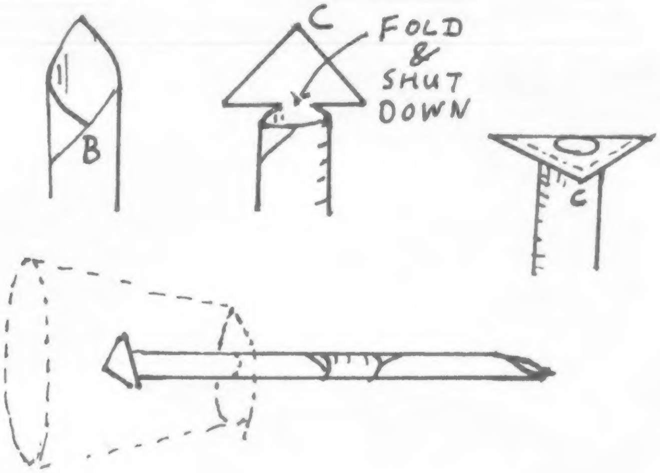
Materials: paper and scissors

Procedure: Take a six to 8½ inch square of paper and fold along one diagonal. Then open out and proceed to roll the paper tightly around a pencil from one end of the diagonal crease to the other end, so that the diagonal rolls along itself as shown in the figure. If a six sided pencil is used, do not wrap too tightly, or the pencil will not come out. When completely rolled it should look like the lower figure. Push the pencil out and paste the last fold at A., or hold it in place with a rubber band or strip of cellotape. The ends of the roll will look like the figure below.





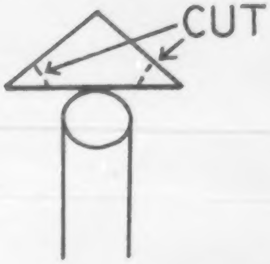
Now from point marked B. at one end cut away on each side, in direction indicated by the small arrows, until the end piece may be opened out into a triangle shape C. The cuts must be at right angles to the main roll and are each a trifle over one third of the circumference of the rolled tube. Now fold the triangle piece at right angles to the tube so that it forms a little cover over the end (see figure below). Trim away a small part of the triangle on each side along the dotted lines indicated in sketch, but do not trim too close. Now place the other end of the tube in your mouth, and, instead of blowing, draw in your breath. This action will cause the little triangular paper lid to vibrate and the instrument will give a bleating sound. The noise can be made louder by rolling a cornucopia or horn and putting on the tube as shown in the last figure, or the tube can be poked through a hole in the bottom of a styrofoam cup, to act in the same way.



Qualitative: How does the reed work?

This type of reed, in which the flap closes off the aperture completely, is known as a "beating reed", and is used in the clarinet, oboe and bassoon. The clarinet has one reed, and the oboe and bassoon two reeds beating together. The vocal cords are also similar.

A puff of air is allowed into the pipe each time the flap opens and closes, and such sharp puffs have a lot of high frequencies in them, in addition to the lowest frequency, or fundamental. Now, cut the corners off the reed, as shown



(Take care that the reed covers the tube--otherwise it will not work). Does the pitch go up or down? The natural frequency of vibration of the reed, (as with all simple harmonic motion) is higher the smaller the mass oscillating.

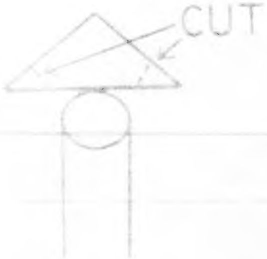
Why does putting the horn over the reed make it sound louder?

In a wind instrument, the reed is not free to vibrate at its natural frequency, as here, but is forced to oscillate at the resonant frequency of the tube of the instrument. The paper reed can only vibrate at a low frequency, so you would need a tube about three feet long to be able to bring the reed into resonance. If you have such a tube, you could try it out.

We can examine the way the voice works using this device. The paper cup over the end of the tube has its own resonant frequency. The vocal tract (larynx, mouth) behaves similarly in the case of the voice. The resonance is at a high frequency, and tends to emphasize frequencies produced by the reed in this vicinity--these resonant frequencies are called formants in the case of the voice, and determine whether you are saying "oo" or "ah", even if our voice holds the same basic fundamental pitch.

While sucking on the reed, close the cup partially with one hand, then open it again. Doing so alters the formants, and it is quite easy to get the device to say "ma ma" or even more difficult vowel sounds with a little practice.

A puff of air is allowed into the tube each time the flap opens and closes, and such sharp puffs have a lot of high frequencies in them, in addition to the lowest frequency, or fundamental. Now, cut the corners off the reed, as shown.



(Take care that the reed covers the tube--otherwise it will not work). Does the pitch go up or down? The natural frequency of vibration of the reed, (as with all simple harmonic motion) is higher the smaller the mass oscillating.

Why does pulling the horn over the reed make it sound louder?

In a wind instrument, the reed is not free to vibrate at its natural frequency, as here, but is forced to oscillate at the resonant frequency of the tube of the instrument. The paper reed can only vibrate at a low frequency, so you would need a tube about three feet long to be able to bring the reed into resonance. If you have such a tube, you could try it out.

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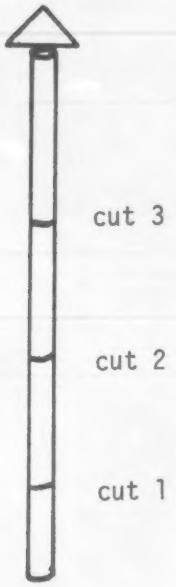
While sucking on the reed, close the cup partially with one hand, then open it again. Doing so alters the formants, and it is quite easy to get the device to say "ma ma" or even more difficult vowel sounds with a little practice.

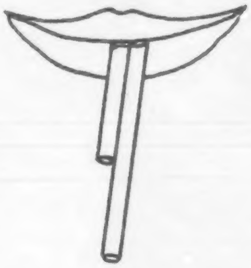
Try cutting the tube shorter, and shorter. Does the pitch of the reed change? At what length does the reed stop functioning? Clearly, the air in the tube is necessary to the functioning of the reed, even though the length does not determine the pitch in the same way as it does for an open pipe.

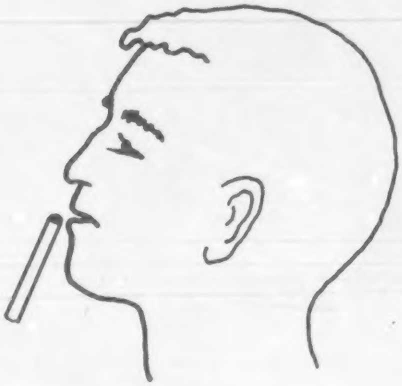


Experiment 64 Combination tones

Materials: - Soda straws

Instructions: - Cut one soda straw two inches long, and one 2 2/3 inches long. One has a pitch approximately 3300 and the other 2828 Hz. . Now, blow them simultaneously as shown





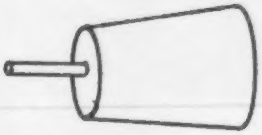
What do you hear?

Quantitative - in addition to the two tones, a third is heard. If you cut a soda straw 8 inches long corresponding to the difference in frequency of the two straws, (825 Hz) it may help you to hear this different tone.

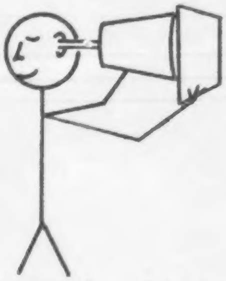
EXPERIMENT AC-5 Resonance in Sound

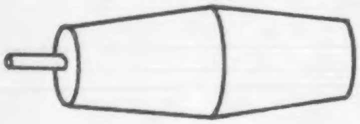
Materials: Two Styrofoam or Dixie cups, straw, sheet of card.

Procedure: Poke a hole with a pencil in the bottom of the cup, and insert about two inches of straw, as shown



Attach it firmly by putting strips of cellotape around the junction. Now carefully insert the straw in your ear, and cover the mouth of the cup with a sheet of card.





Leave an opening for sound to get in as shown. You have probably, at some time or another, "listened to the sea" by holding a conch shell to your ear, and you will find you can hear the sea just as clearly with the cup as the shell. What is happening is that the cup resonates to a certain unique pitch or frequency. Just as the pendulum responds to oscillations of only one frequency, so the cup responds to only one pitch. So, of all the sounds in the room, you hear this pitch greatly exaggerated. To find which pitch this is, ask someone to sing or hum continuously from low to high. You will hear the resonant pitch stand out, sounding loud compared with the others. Now, the size of the cup determines the pitch, so take a second cup, and put it over the first instead of the card with enough gap between to let sound in (or you can poke a hole through the base). You will find it resonates at a lower frequency and, in fact, it will be about an octave lower, because you have twice the volume of container. You can also vary the resonant pitch by moving the card across the top. When the card covers most of the cup, the resonance is sharp, (i.e., that one pitch is much exaggerated, or amplified) as the card is moved back, the resonance pitch becomes less sharp, until, when completely removed, there is practically no resonance at all.

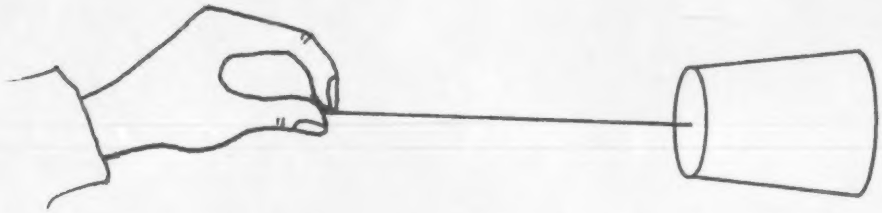
Qualitative questions: Why should a hollow container resonate in this way? Think about the way sound bounces around inside the cup, and the way, if you blow across the top of a bottle (or across the opening left when the card all but covers the cup) you get a note, which is the resonant frequency of the device--this can be used to find the resonant frequency of your cup-resonator instead of humming.

Quantitative questions: The linear size of the cup is a fraction of the wavelength to which it resonates. Measure the size of the cup. The frequency times the wavelength is the velocity of sound, so to what frequency does the size correspond? The velocity of sound in air is 1100 ft. per second. Is the real frequency higher or lower than this? (Remember, middle C on a piano has a frequency of about 260 cycles per second, and when you blow across a straw 6 inches long, open at both ends, it vibrates at 1100 cycles per second.

Experiment 16 The string Telephone

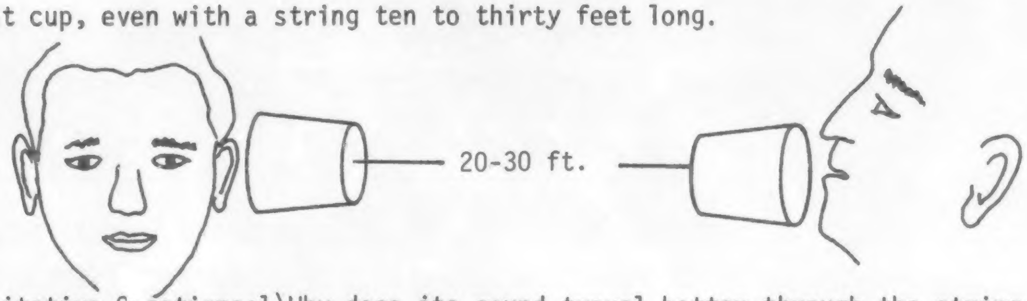
Materials: two styrofoam cups, string

Procedure: Tie a knot in a piece of string, and draw the string through a small hole in the bottom of a Dixie or styrofoam cup. Draw the string between the thumb and forefinger.



Qualitative Questions: Are the vibrations generated longitudinal or transverse? Why does the cup appear to amplify the sound? Does the pitch go up and down as you hold the string tighter, and put more tension on the string?

Now, push the end of the string through a small hole in the bottom of the second cup. When you talk into it, the speech is clearly audible in the first cup, even with a string ten to thirty feet long.



Qualitative Questions:1)Why does its sound travel better through the string than through the air?2)Are high or low frequencies transmitted better? 3)Does this depend on the tightness and density of the string and the size of the cup? 4) Approximately how much more energy reaches the hearer's ear with the cup than without it?

Flying circus of physics 1.9